# A Generalized Perks Formula for Old-Age Mortality

Sten Martinelle

SCB

R&D Report
Statistics Sweden
Research - Methods - Development
U/STM - 38

INLEDNING

TILL

**R & D report : research, methods, development, U/STM / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1987. – Nr 29-41.**

## Föregångare:

Promemorior från U/STM / Statistiska centralbyrån. – Stockholm : Statistiska centralbyrån, 1986. – Nr 25-28.

## Efterföljare:

R & D report : research, methods, development / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1988-2004. – Nr. 1988:1-2004:2.

Research and development : methodology reports from Statistics Sweden. – Stockholm : Statistiska centralbyrån. – 2006-. – Nr 2006:1-.

R & D Report, U/STM 1987:38. A generalized perks formula for old-age mortality / Sten Martinelle.
Digitaliserad av Statistiska centralbyrån (SCB) 2016.

urn:nbn:se:scb-RnD-USTM-1987-38

# A Generalized Perks Formula for Old-Age Mortality

Sten Martinelle

SCB

R&D Report
Statistics Sweden
Research - Methods - Development
U/STM - 38

| | |
|---|---|
| Från trycket | Oktober 1987 |
| Producent | Statistiska centralbyrån, Enheten för statistiska metoder |
| Ansvarig utgivare | Staffan Wahlström |
| Förfrågningar | Sten Martinelle, tel. 08 7834377 |


ISSN 0283-8680
Printed in Sweden
Garnisonstryckeriet, Stockholm 1987

1987-08-17

# A Generalized Perks Formula for Old-Age Mortality

By Sten Martinelle

**Contents** **page**



**Appendix**

**Abstract**

A heterogenous cohort with intensity functions of individuals following the Makeham law is considered. It is shown that if the frailty variable obeys a generalized form of the gamma distribution then the total force of mortality can be seen as a generalization of Perks formula. The model is applied to mortality of centenarians in Sweden and other countries.

**Keywords**

Old-age mortality, centenarians, heterogeneity, frailty, gamma distribution, Perks formula

## 1. Introduction

Adult mortality, measured by the force of mortality

$$\mu(x) = -\frac{dl(x)}{dx} / l(x) \qquad (1.1)$$

where l(x) is the number of survivors of a cohort at age x, is well approximated in broad age intervals by the Gompertz-Makeham formula

$$\mu(x) = A + Be^{kx} \quad (A \geq 0, \text{ B and k} > 0) \qquad (1.2)$$

It is well known that the exponential increase is slowed down at higher ages. Perks (1932) found that the logistic curve

$$\mu(x) = \frac{A + Be^{kx}}{1 + De^{kx}} \quad (A \geq 0, \text{ B, D and k} > 0) \qquad (1.3)$$

gave a much better fit to adult and old age mortality. For a review of these theories see Spiegelman (1968, Chapt. 6).

In the discussion of Perks paper Trachtenberg advances the idea of a heterogeneous population: Individuals follow the Gompertz law (A = 0 in (1.2)) but with varying parameters B and k. Beard (1959) showed that if B (which measures inability to withstand destruction, called "frailty" by Vaupel et al.(1979)) in such a heterogenous cohort is gamma distributed at birth, then the totalforce of mortality is given by Perks formula.

Perks formula implies that $\mu(x)$ approaches a constant limit value in high ages. Vincent (1951), Depoid (1973), Thatcher (1981) and Barrett (1985) have studied the mortality of centenarians. Although Barrett finds a decline in estimated male mortality after age 104, there seems to be no evidence for a plateau in the mortality rates for the centenarians. The decline observed by Barrett is based on only 23 men aged 105 and no decline is found in the female rates.

Horiuchi and Coale (1983) analyse the age dependence of female mortality in several modern industrialized countries using the age-specific rate of mortality change with age, i.e. the derivative $k(x) = d\ln(\mu(x))/dx$. The age interval is 50-95 years. They find that the function k(x) has a peak about age 75 and its peak is close to 0.12 . This pattern supports the Perks model and "estimated k(x) values derived by fitting the Perks model to the logarithms of age-specific death rates agree quite well with the smoothed sequence of observed k(x) values, except for some departures at both ends." In the higher ages the observed k(x) values do not approach zero as predicted by the Perks model.

Using a micro simulation model Qvist (1987) models the mortality in coronary heart disease, the dominant cause of mortality at high ages. As a "frailty" index he uses the level of atherosclerosis leisons. The distribution of the frailty index is based on an investigation by Rissanen (1972) concerning patients who have suffered violent deaths. The approach is interesting for at least two reasons: (1) The heterogeneity assumption is medically well established and (2) The distribution of the frailty index is markedly skewed.

In modelling the total mortality at old ages we think the heterogeneity assumption is essential. Variation in genetic constitution and life style account for differences between individual mortality rates. We think, however, that the assumptions leading to Perks formula should be somewhat modified since the plateau predicted by the formula does not seem to exist. Therefore we change the assumption on the frailty distribution in the neighbourhood of zero. We move the distribution to the right. Instead of the gamma distribution itself we use a generalized or shifted gamma distribution (Hahn and Shapiro (1967) , p 89).

## 2. A model of the frangible man

We will model the total force of mortality in a cohort of men or women. To each individual corresponds a positive value of a frailty variable z and the value is supposed to be the same during the total lifetime. The survival and intensity function for an individual of frailty z is denoted $l(x|z)$ and $\mu(x|z)$ respectively. The frequency function of the frailty variabel at age x is denoted $f(z|x)$ and for simplicity of notation we put $g(z) = f(z|x=0)$, i.e. $g(z)$ is the frequency function at birth. The frailty distribution at birth seems to be of little relevance for old-age mortality and we shall soon see that the time of birth is just a practical starting point. We can now state the following theorem on the total force of mortality $\mu(x)$.

**Theorem** ("The theorem of the frangible man")

If the intensity functions of individuals follow the Makeham law:

$$\mu(x|z) = A + ze^{kx} \quad (A \geq 0,\ k \geq 0) \tag{2.1}$$

and the frailty variable has a generalized (i.e. shifted) gamma distribution, with density

$$g(z) = \begin{cases} \dfrac{b^a (z-c)^{a-1}}{\Gamma(a)} e^{-b(z-c)} & \text{for } z > c \\ & (a,\ b, \text{ and } c > 0) \\ 0 \quad \text{for } z \leq c & \end{cases} \tag{2.2}$$

with parameters a,b och c, then the total force of mortality $\mu(x)$ can be written:

$$\mu(x) = \frac{A + Be^{kx}}{1 + De^{kx}} + c\, e^{kx} \tag{2.3}$$

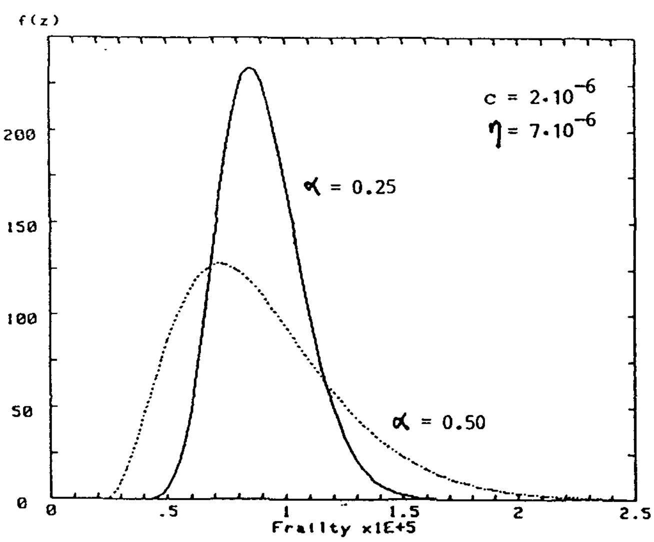
where $B = (A + ak)/(kb-1)$ och $D = 1/(kb-1)$.

The proof is given in appendix A (A1).

The proof also shows that the frailty variable has a generalized gamma distribution at each age x; cf (A1.5). In describing the old-age mortality, we actually only need the frailty distribution assumption at age 85, say. If we assume that the frailty variable at that age has a generalized gamma distribution and that $\mu(x|z)$ for $x > 85$ follows the Makeham law (2.1), then the formula (2.3) is still applicable.

The equation (2.3) shows the close connection to the Perks formula. The additional term is due to our assumption about the "frangible man" - the frailty variable cannot take on values too close to zero. As a consequence $\mu(x) \to +\infty$ as $x \to +\infty$ , which we think is a plausible property of the total force of mortality $\mu(x)$.

Fig 1. Examples of frailty distributions



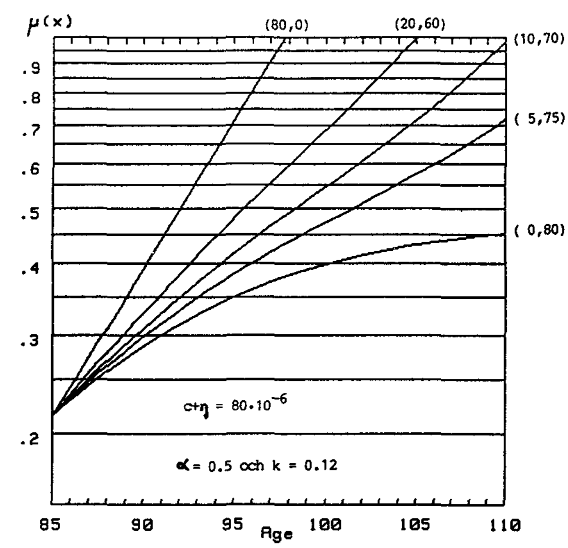

The frailty $Z = c + U$ has the lower bound c. The deviation from this bound, U, is gamma distributed with expectation $\eta$ and relative standard deviation $\alpha$.

What the equation (2.3) does not show is the close connection to the form of the frailty distribution. Assuming A = 0, which at high ages seems a reasonable approximation, equation (2.3) can be written (Appendix, A2):

$$\mu(x) = \left\{ c + \frac{\eta}{1 + \eta\alpha^2 \int_{x_0}^{x} e^{kt} dt} \right\} e^{kx} \qquad (2.4)$$

where $\eta$ is the mean of U = Z - c, the deviation of the frailty variable from the lower bound z = c, and $\alpha$ is the relative standard deviation of U. The measure $\alpha$ is directly related to the skewness of the distribution (Hahn and Shapiro(1967), p 124). See fig 1.

Fig 2. The effect of changing the ratio $c/(c+\eta)$ in (2.4)




Extreme cases are: $\mu_U(x) = me^{kx}$ and

$$\mu_L(x) = \frac{m}{1 + mI(x)} e^{kx}, \qquad \text{where } I(x) = \alpha^2 \int_{85}^{x} e^{kt} dt.$$

From (2.4) follows immediately $\mu(x_0) = (c+\eta)e^{kx_0}$ . Thus for given k and $\alpha$, all curves with the sum $c + \eta = m$ pass through the same point $(x_0,\mu(x_0))$ as shown in fig 2 for $\alpha = 0.5$, $k = 0.12$, $m = 8x10^{-7}$ and $x_0 = 85$. The extreme cases $\eta = 0$ and $c = 0$ correspond to Gompertz and Perks law (with A = 0) respectively.

## 3. Fitting the model to the data

### 3.1 The data

Extensive data on old-age mortality for four countries (France, Sweden, Switzerland and The Netherlands) and several periods are given by Depoid(1973). To these data we have added mortality data for Sweden during later years (see appendix B). In this way we have disposed of mortality patterns as wide apart as Switzerland 1876- 1914 and Sweden 1979-84.

One year risks of death ($q_x$) have been transformed to intensities, using the formula:

$$\mu(x+0.5) \approx -\ln(1-q_x) \qquad (x=85,86,\ldots.) \qquad (3.1)$$

For each country, period and sex we have such a set of estimated intensities. The number of observations in each set is about 20. Intensities based on less than 12 survivals have been excluded. By this choice we have avoided the cases $q_x = 0$ and $q_x = 1$ .

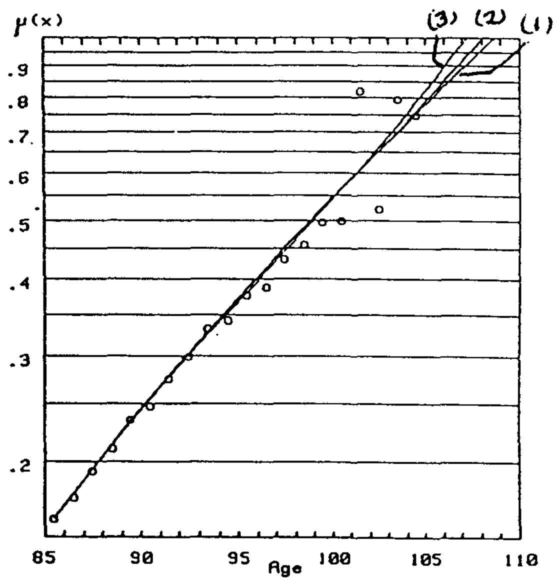
The data are not purely period data. Thus, in the set of intensities for Sweden 1967-73 the cohort born 1877 is included, having their 90 years birthday during the first year of the period, and then followed during six years until the age of 96 in 1973. In this way the mortality rate of persons aged 90 is based on the generations born 1877-1882, while the rate of persons aged 91 is calculated from the mortality of the cohorts 1876-1881. As usual, age and cohort effects are not separable - a decrease in cohort mortality will show up as an increase in the age effect. But even cohort data are problematic - a period effect will be confounded with the age effect.

### 3.2 Some additional assumptions

Fitting the model (2.4) to our mortality data cannot unambiguously determine the four parameters. Unless additional information on the parameters is supplied, a whole variety of parameter values will give about as good a fit as possible. This is true also of the Perks model. Perks(1932) observes, fitting the curve to the mortality rates at ages 40-100 years, that the parameter k could be changed without essentially affecting the goodness of fit. As we start at age 85 ($x_0$ = 85), the data contain less information about the curvature of the mortality rates, which also means less information about the frailty distribution.

This lack of information is illustrated in fig 3. Using the least-squares method, we have fitted equation (2.4) to the mortality intensities of Swedish men 1979-84 for three different combinations of $\alpha$ and k. It is seen that the predicted intensities for the different combinations are almost identical at ages 85-105.

Fig 3. All the parameters cannot be estimated




Data: Sweden 1979-84, men

The function (2.4) has been fitted to the data for three different combinations of α and k: (1) α=0.4 , k=0.11 (2) α=0.5, k=0.12 and (3) α=0.6, k=0.13

The pairs of α and k used in fig 3 (and all pairs on the same line in the neighbourhood of k = 0.12) almost minimize the total sum of squares taken over all our data sets. To solve the problem of indeterminacy we have chosen α = 0.5 and k = 0.12 as "universal" constants to be used in the following. To this α-value corresponds a rather skew frailty distribution (fig 1) - and we think the distribution should be rather skew (Qvist(1987)) - and the k-value we think is in good agreement with the findings of Horiuchi and Coale(1983). But the exact choice must of course be somewhat arbitrary.

Analysing all data sets by means of the same $\alpha$ and k simplifies the comparison of mortality patterns between countries and periods as we get a common basis for the parameters. Thus, the estimated intensity level at age x is directly related to the sum $m = c + \eta$ and, if comparing two data sets with $m_1$ and $m_2$ respectively and $m_1 > m_2$, then we have $\hat{\mu}_1(x_0) > \hat{\mu}_2(x_0)$. Moreover, the ratio c/m contains the information of the form of the curve (cf fig 2).

### 3.3 Least-squares estimation of the parameters

Given $\alpha$ (= 0.5) and k (= 0.12), the force of mortality (2.4) is a function of the two parameters c and $\eta$. To fit the model to observed intensities $\tilde{\mu}(x)$ (x=85.5,..), we will use the method of least-squares. Since the function $\mu(x)$ is nonlinear in $\eta$, we replace it with a linear approximation in a neighbourhood of a guessed value of $\eta$, denoted $\eta_0$ (Draper and Smith(1981), Chap.10). Then we get the following regression equation, linear in the two unknown parameters c and $\Delta\eta$ ($\Delta\eta = \eta - \eta_0$):

$$\tilde{\mu}(x) = \{c + \eta_0/(1+\eta_0 I(x)) + \Delta\eta/(1+\eta_0 I(x))^2\}e^{kx} + \varepsilon \qquad (3.3)$$

where $I(x) = \alpha^2(e^{kx} - e^{85.5k})/k$ .

Assuming the number of deceased aged x binomially distributed with parameters $n_x$ and $q_x$, then, if the model is true, the variance of the residual is approximately:

$$\mathrm{Var}(\varepsilon|x) \approx n_x^{-1}q_x/(1-q_x) \qquad (3.4)$$

The number of survivals at age x, denoted $n_x$, decreases rapidly with x - the number of survivals at age 100 is just a small fraction of the number at age 85. This fact means that the estimates $\tilde{\mu}(x)$ are of very different precision. Therefore, we use weighted least-squares with weights $w_x^2 = n_x(1-q_x)/q_x$, i.e. we estimate c and $\Delta\eta$ by minimizing the sum of squares:

$$Q = \Sigma\, w_x^2\{\tilde{\mu}(x) - (c + \eta_0/(1+\eta_0 I(x)) + \Delta\eta/(1+\eta_0 I(x))^2)e^{kx}\}^2 \quad (3.5)$$

$x=85.5,\ 86.5,\ \ldots.$ and $w_{x+.5} = w_x$ . If $n_x<12$ then $w_x = 0$ .

The sum of squares can also be written:

$$Q = \Sigma\{y_x - \beta_1 g_1(x) - \beta_2 g_2(x)\}^2 \quad (3.6)$$

where $y_x = w_x\{\tilde{\mu}(x) - \eta_0 e^{kx}/(1+\eta_0 I(x))\}$ , $g_1(x) = w_x e^{kx}$ ,

$g_2(x) = w_x e^{kx}/(1+\eta_0 I(x))^2$ , $\beta_1 = c$ och $\beta_2 = \Delta\eta$ .

With this change of notation we are back to the ordinary linear regression model, so the coefficents $\beta_1$ and $\beta_2$ can be determined by a standard program for regression analysis. Note, however, that there is no constant term ($\beta_0 = 0$). From the estimates $\hat{\beta}_1$ and $\hat{\beta}_2$ we then get $\hat{c} = \hat{\beta}_1$ and $\hat{\eta} = \eta_0 + \hat{\beta}_2$.

Substituting the estimate of $\eta$ for $\eta_0$, the procedure can be repeated an arbitrary number of times. If we choose the value of $\eta_0$ in the following way, then just the first step in this iteration procedure seems to be sufficient:
(1) Put $m_0 = c_0 + \eta_0 = \tilde{\mu}(85.5)e^{-85.5k}$, which means that the intensity function containing $c_0$ and $\eta_0$ equals the estimated intensity at age $x = 85.5$ . (2) Put $\eta_0 = 0.75m_0$ and $c_0 = m_0 - \eta_0$ .

The weights $w_x$ have been computed from these initial values of c and $\eta$.

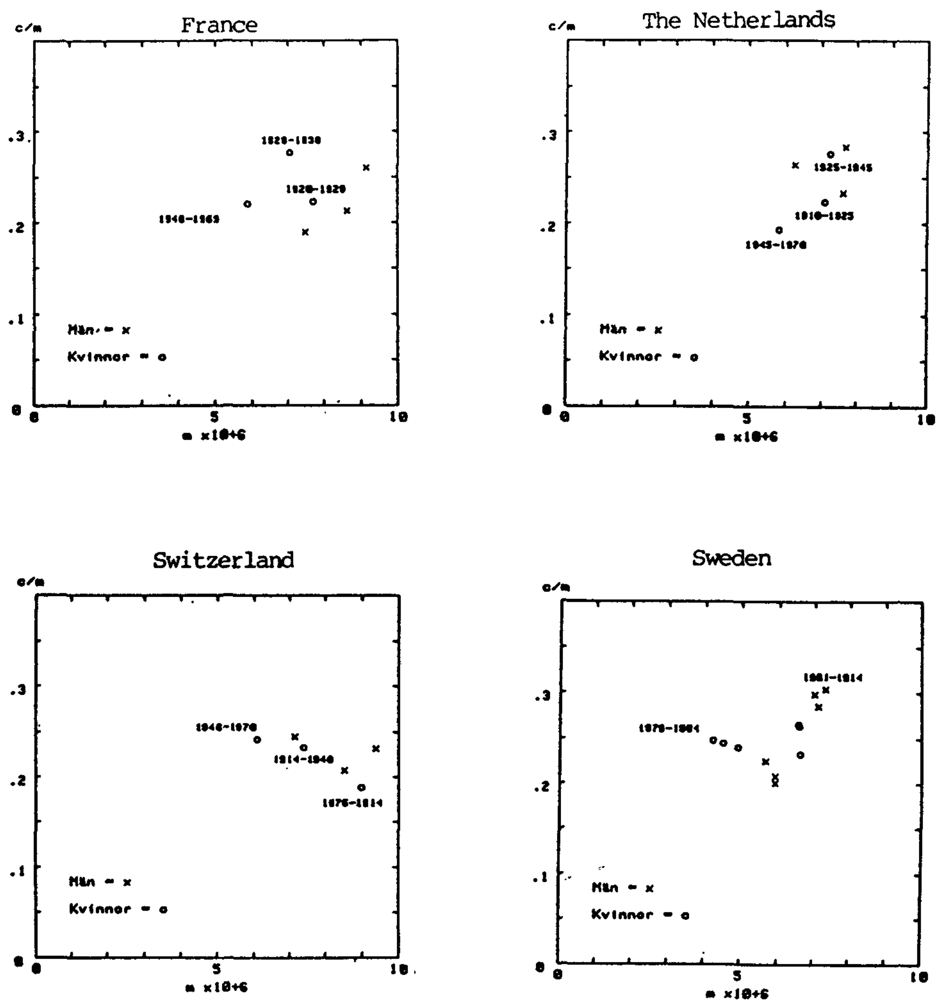
### 3.4 The parameter estimates

The parameter estimates are given in table 1 below. See also fig 4. The estimates are given as the sum $\tilde{m} = \tilde{c} + \tilde{\eta}$ and the ratio $\tilde{c}/\tilde{m}$ and refer to the frailty distribution at age 85 (cf fig 1) with $\alpha$=0.5 .

**Table 1. Estimates of expected frailty at age 85 ($m = c + \eta$) and the lower bound of the frailty distribution in percent of expected frailty (c/m). Twice the st.dev. is given for each estimate($2\sigma$).**

| | | Men | | | | Women | | | |
|---|---|---|---|---|---|---|---|---|---|
| Country | Period | m ($x10^6$) | $2\sigma$ | c/m (%) | $2\sigma$ | m ($x10^6$) | $2\sigma$ | c/m (%) | $2\sigma$ |
| France | 1920-29 | 9.2 | .10 | 26 | 3 | 7.8 | .08 | 22 | 2 |
| " | 1929-38 | 8.7 | .10 | 20 | 3 | 7.1 | .10 | 26 | 4 |
| " | 1948-69 | 7.6 | .08 | 21 | 3 | 5.9 | .02 | 23 | 1 |
| The Netherl. | 1910-25 | 7.8 | .14 | 21 | 4 | 7.1 | .14 | 22 | 5 |
| " | 1925-45 | 7.7 | .12 | 27 | 5 | 7.1 | .10 | 28 | 4 |
| " | 1945-70 | 6.4 | .06 | 26 | 3 | 5.9 | .04 | 19 | 2 |
| Switzerland | 1876-14 | 9.5 | .16 | 23 | 5 | 9.0 | .18 | 19 | 5 |
| " | 1914-48 | 8.6 | .10 | 21 | 3 | 7.5 | .12 | 22 | 4 |
| " | 1948-70 | 7.3 | .10 | 23 | 4 | 6.2 | .06 | 23 | 3 |
| Sweden | 1901-14 | 7.3 | .16 | 30 | 8 | 6.6 | .08 | 26 | 4 |
| " | 1914-30 | 7.1 | .10 | 29 | 4 | 6.6 | .08 | 23 | 3 |
| " | 1930-45 | 7.1 | .10 | 29 | 4 | 6.6 | .06 | 26 | 3 |
| " | 1945-67 | 6.7 | .06 | 29 | 3 | 6.2 | .06 | 23 | 2 |
| " | 1967-73 | 6.0 | .04 | 20 | 2 | 4.9 | .06 | 23 | 4 |
| " | 1973-79 | 5.9 | .08 | 19 | 3 | 4.5 | .04 | 23 | 3 |
| " | 1979-84 | 5.6 | .10 | 23 | 4 | 4.1 | .08 | 27 | 5 |

Fig 4. Parameter estimates




Expected frailty at age 85 on the horizontal axis (m = c + η) and the lower bound of the frailty distribution in percent of the expected frailty (c/m) on the vertical axis.
The c-value is about 25% of the m-value.

The sum m indicates the mortality level at age 85.5:

$$\hat{\mu}(85.5) = me^{85.5k} \text{ where } k = 0.12 \text{ .}$$

Example: For men in France during 1920-29 we have $m = 9.2 \times 10^{-6}$ and, therefore, $\hat{\mu}(85.5) = 0.2628$ or $\hat{q}_{85} \approx 1 - e^{-.2628} = 0.2311$ .

The indicator of the mortality level, m, varies systematically between sexes, countries and periods. The ratio c/m seems on the other hand to fluctuate more arbitrarily around a constant mean of about 0.24. This is the reason why we express c as a proportion of m. But the total variation of the ratio c/m does not seem to be due to the random variation in the estimated intensities. Some systematic differences seem to exist.

### 3.5 The fit of the model

We have plotted estimated and predicted mortality rates ($q_x$). See appendix C. No systematic pattern recurring in several data sets have been found. We have also tested the goodness of fit by a $\chi^2$-test. The calculated $\chi^2$-value is printed on the top of each figure.

The $\chi^2$-test is based on the sum of squared residuals: From the definition of the weights $w_x$ it follows that the variance of $y_x$ in (3.6) equals almost unity. Since $y_x$ is a linear function of the intensities $\tilde{\mu}(x)$, which are approximately normally distributed and independeant for large $n_x$, the sum of squared residuals is approximately distributed as $\chi^2$ with $f = N-2$ degrees of freedom, where N is the number of terms in the sum of squares (3.6).

The test values are found in table 2. In the case of a significant value ($p<0.05$), then the sum of squares has been split into two parts: One over the interval 85-94 years and the other over the ages above 94 years.

**Table 2. $\chi^2$-test of goodness of fit**

| Country | Period | Men | | | | Women | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\chi^2$ | | | df | $\chi^2$ | | | df |
| | | (1) | (2) | (1)+(2) | | (1) | (2) | (1)+(2) | |
| France | 1920-29 | 9.4 | 21.6 | 31.0 | 16 | 10.9 | 23.9 | 34.8 | 17 |
| " | 1929-38 | 10.8 | 25.3 | 36.1 | 16 | 14.3 | 60.3 | 74.6 | 17 |
| " | 1948-69 | 32.5 | 67.2 | 99.7 | 19 | 19.3 | 19.7 | 39.0 | 21 |
| The Neth. | 1910-25 | | | 14.2 | 16 | | | 26.2 | 16 |
| " | 1925-45 | | | 19.8 | 16 | | | 21.3 | 17 |
| " | 1945-70 | | | 17.1 | 18 | | | 19.1 | 20 |
| Switzerl. | 1876-14 | | | 11.8 | 15 | | | 19.1 | 15 |
| " | 1914-48 | | | 8.1 | 16 | | | 20.2 | 16 |
| " | 1948-70 | | | 14.1 | 16 | | | 19.0 | 18 |
| Sweden | 1901-14 | | | 19.8 | 14 | | | 14.0 | 17 |
| " | 1914-30 | | | 15.0 | 16 | | | 18.3 | 17 |
| " | 1930-45 | | | 16.7 | 17 | | | 10.7 | 18 |
| " | 1945-67 | | | 15.0 | 17 | | | 19.6 | 19 |
| " | 1967-73 | | | 3.6 | 17 | | | 16.8 | 19 |
| " | 1973-79 | | | 16.0 | 18 | | | 13.4 | 19 |
| " | 1979-84 | | | 18.7 | 18 | 21.2 | 15.5 | 36.7 | 20 |

(1) = The sum of squared residuals 85 - 94 years and (2) >95 years
(1)+(2) is, if the model is correct, distributed as $\chi^2$ with df degrees of freedom.

According to the $\chi^2$-test, the fit of the model is less good in the case of French data and the partial sums of the squared residuals show that the largest contribution to the test values comes from the extremely old. No distinct systematic pattern can be seen, however. One explanation to significant test values for France is the fact that the French data are based on larger populations and, therefore, the probability of detecting even small discrepancies is increased.

How should we explain the significant $\chi^2$-values? In the first place we have of course questioned the form of the intensity function. A visual inspection of observed and predicted death risks has not revealed, however, any significant pattern common to the data sets. Therefore, it seems more probable that the cause is to find somewhere else. In fact several of the reasons below may contribute to the significant $\chi^2$-values:

- False significances. In a sequance of 32 independeant tests on the 5% level 1.6 false significances are expected and the probability of the occurrence of two or more is almost 50%.

- The normal approximation is less good for intensities based on small groups. Therefore, the $\chi^2$-approximation is supposed to be less adequate at extremly high ages.

- The approximation of the residual variance is based on large groups and is less adequate at extremly high ages.

- The residuals are not independent due to cohort effects.

- The quality of the data.

## 4. Application to the Swedish life table

The Swedish life tables are based on observed risks of death up to the age 90 years. From the age 91 years the risks are estimated according to Wittsteins formula: $q_x = a^{-(M-x)^n}$ , where M is an assumed maximal lifespan (105 years for men and 106 for women), a and n are constants determined from the data.

We have recomputed the table 1981 - 85 using the generalized Perks formula (table 3 a and b). The expected number of centenarians are more than doubled according to our calculation.

**Table 3a. Comparison of methods of smoothing.**
**Swedish men 1981 - 85.**

| Age | Risks of death [1] (0/00) | | | Number alive ($10^5$ at birth) | | Life exp. (years) | |
|---|---|---|---|---|---|---|---|
| (x) | $\tilde{q}_x$ | $\tilde{q}_x^N$ | $\tilde{q}_x^W$ | $l_x^N$ | $l_x^W$ | $e_x^N$ | $e_x^W$ |
| 91 | 239 | 236 | 236 | 6100 | 6100 | 3.10 | 3.01 |
| 92 | 257 | 254 | 255 | 4662 | 4662 | 2.90 | 2.78 |
| 93 | 282 | 272 | 277 | 3479 | 3471 | 2.71 | 2.56 |
| 94 | 291 | 292 | 301 | 2532 | 2509 | 2.54 | 2.36 |
| 95 | 320 | 311 | 328 | 1794 | 1753 | 2.38 | 2.41 |
| 96 | 328 | 332 | 358 | 1235 | 1178 | 2.24 | 2.16 |
| 97 | 342 | 353 | 392 | 825 | 756 | 2.10 | 1.96 |
| 98 | 335 | 374 | 429 | 534 | 460 | 1.97 | 1.78 |
| 99 | 415 | 397 | 472 | 334 | 263 | 1.85 | 1.61 |
| 100 | - | 419 | 521 | 202 | 139 | 1.74 | 1.44 |
| 101 | - | 443 | 577 | 117 | 66 | 1.63 | 1.28 |
| 102 | - | 467 | 645 | 65 | 28 | 1.53 | 1.12 |
| 103 | - | 492 | 725 | 35 | 10 | 1.44 | .97 |
| 104 | - | 518 | 829 | 18 | 3 | 1.35 | .82 |
| 105 | - | 544 | 1000 | 9 | - | 1.27 | .67 |
| 106 | - | 572 | - | 4 | - | 1.19 | .50 |
| 107 | - | 600 | - | 2 | - | 1.10 | - |
| 108 | - | 629 | - | 1 | - | 1.00 | - |
| 109 | - | 658 | - | - | - | .84 | - |
| 110 | - | 688 | - | - | - | .50 | - |

1) Notation: $\tilde{q}_x$ = observed risk of death= $D_x/(M_x + d_x)$ , where $D_x$ is the number of deceased during the period in the age interval x to x+1, $d_x$ is the the number of deceased during the period in the same age interval but after the birthday and $M_x$ is the mean population aged x.

$\tilde{q}_x^N$ = smoothed risk of death, new formula and

$\tilde{q}_x^W$ = " " " " , Wittsteins formula.

**Table 3b. Comparison of methods of smoothing.
Swedish women 1981 - 85.**

| Age | Risks of death[1) ] (0/00) | | | Number alive ($10^5$ at birth) | | Life exp. (years) | |
|---|---|---|---|---|---|---|---|
| (x) | $\tilde{q}_x$ | $q_x^N$ | $q_x^W$ | $l_x^N$ | $l_x^W$ | $e_x^N$ | $e_x^W$ |
| 91 | 194 | 192 | 194 | 16059 | 16059 | 3.69 | 3.47 |
| 92 | 209 | 208 | 214 | 12976 | 12936 | 3.44 | 3.18 |
| 93 | 230 | 225 | 236 | 10273 | 10166 | 3.22 | 2.91 |
| 94 | 242 | 243 | 260 | 7959 | 7768 | 3.01 | 2.66 |
| 95 | 257 | 262 | 287 | 6025 | 5746 | 2.81 | 2.41 |
| 96 | 293 | 281 | 318 | 4449 | 4095 | 2.63 | 2.19 |
| 97 | 298 | 301 | 351 | 3200 | 2795 | 2.46 | 1.97 |
| 98 | 315 | 321 | 389 | 2239 | 1813 | 2.31 | 1.77 |
| 99 | 323 | 342 | 432 | 1520 | 1107 | 2.16 | 1.58 |
| 100 | - | 364 | 481 | 1000 | 628 | 2.03 | 1.40 |
| 101 | - | 386 | 536 | 636 | 326 | 1.90 | 1.23 |
| 102 | - | 409 | 599 | 390 | 151 | 1.78 | 1.07 |
| 103 | - | 433 | 671 | 231 | 61 | 1.67 | .92 |
| 104 | - | 457 | 756 | 131 | 20 | 1.57 | .78 |
| 105 | - | 483 | 859 | 71 | 5 | 1.46 | .64 |
| 106 | - | 508 | 1000 | 37 | 1 | 1.36 | .50 |
| 107 | - | 535 | - | 18 | - | 1.25 | - |
| 108 | - | 563 | - | 8 | - | 1.12 | - |
| 109 | - | 591 | - | 4 | - | .91 | - |
| 110 | - | 620 | - | 2 | - | .50 | - |

1) Notation: $\tilde{q}_x$ = observed risk of death= $D_x/(M_x + d_x)$ , where $D_x$ is the number of deceased during the period in the age interval x to x+1, $d_x$ is the the number of deceased during the period in the same age interval but after the birthday and $M_x$ is the mean population aged x.

$\tilde{q}_x^N$ = smoothed risk of death, new formula

$\tilde{q}_x^W$ = " " " " , Wittsteins formula

## 5. References

Barett,J.C.(1985):The Mortality of Centenarians in England and Wales. Arch. Gerontol.Geriatr., 4, p 211-218 .

Beard,R.E.(1959):Note on Some Mathematical Mortality Models, Colloquia on Ageing, CIBA Foundation, Vol 5. Churchill, London

Beard,R.E.(1961):A Theory of Mortality Based on Actuarial, Biological and Medical Considerations. IUSSP 1961, New York , Vol 1, p 611-25.

Depoid,F. (1973):La mortalité des grands vieillards. Population 28, p 755-792

Draper,N.R. and Smith H. (1981):Applied Regression Analysis Sec.ed., Wiley.

Hahn,G.J, and Shapiro,S.S.(1967):Statistical Models in Engineering. Wiley.

Horiuchi,S. and Coale,A.J.(1983):Age Patterns of Mortality for Older Women: An Analysis Using the Age-Specific Rate of Mortality Change with Age. The 1983 Annual Meeting of the Population Association of America Pittsburgh, Pennsylvania. (stencil)

Horiuchi,S.(1983):The Long-Term Impact of War on Mortality: Old-Age Mortality of the First War Survivors in the Federal Republic of Germany. Population Bulletin of the United Nations, No. 15, p 80-92

Manton,G. et al. (1986):Alternative Models for the Heterogenity of Mortality Risks Among the Aged. JASA 81, p 635-644

Perks,W. (1932): On Some Experiments in the Graduation of Mortality Statistics. Journal of the Institute of Actuaries. Vol LXIII, 1932, p 12-57 .

Qvist,J.(1987) : A Micro Simulation Model for Mortality. Statistics Sweden (stencil).

Redington,F.M.(1969):An Exploration into Patterns of Mortality. Journal of the Institute of Actuaries, Vol 95, Part II, No 401, p 243-317

Rissanen,V.(1972):Aortic and Coronary Atherosclerosis in a Finnish Autopsy Series of Violent Deaths. Annales Academiae Scientiarum Fenniace, Series A, V.Medica, 155 .

Spiegelman,M.(1968):Introduction to Demography, Cambridge, MA., Harvard University Press.

Thatcher,A.R.(1981):Centenarians. Population Trends 25, p 11-14

Vaupel,J.W., Manton,K.G. and Stallard,E.(1979):The Impact of Heterogeneity in Individual Frailty on the Dynamics of Mortality. Demography,Vol 16, No 3, p 439-454

Vincent,P.(1951):La mortalité des vieillards. Population, No 2, p 181-204

## Appendix A

### A1. Proof of (2.3)

The relation between $\mu(x)$ and $\mu(x|z)$ is (Beard (1959, Appendix)):

$$\mu(x) = \int_0^\infty \mu(x|z)f(z|x)dz \tag{A1.1}$$

With the assumption (2.1) we get:

$$\mu(x) = A + e^{kx} \int_0^\infty z\ f(z|x)dz = A + e^{kx}E\{Z|x\} \tag{A1.2}$$

But $f(x|z) = g(z)l(x|z)/l(x)$ where $l(x|z)$ is determined by (2.1):

$$l(x|z) = e^{-\int_0^x \mu(u|x)du} = e^{-Ax}\ e^{-z(e^{kx}-1)/k} \tag{A1.3}$$

Then $g(z)l(x|z) = H(x)(z-c)^{a-1}e^{-zb(x)}$ where $H(x)$ does not contain $z$ and

$$b(x) = b + (e^{kx}-1)/k. \tag{A1.4}$$

Further $\quad l(x) = \int_c^\infty g(z)l(x|z)dz$ and therefore

$$f(z|x) = \frac{(z-c)^{a-1}e^{-zb(x)}}{\int_c^\infty (z-c)^{a-1}e^{-zb(x)}dz} =$$

$$= \frac{(z-c)^{a-1}e^{-b(x)(z-c)}}{\int_c^\infty (z-c)^{a-1}e^{-b(x)(z-c)}dz} \tag{A1.5}$$

which means a generalized gamma distribution with parameters a, b(x) and c.

But then, since Z-c is gamma distributed with parameters a and b(x), we have (Hahn and Shapiro (1967), p 124)

$$E\{Z|x\} = c + E\{Z-c|x\} = c + a/b(x) \tag{A1.5}$$

Inserting this expression for $E\{Z|x\}$ in (A1.2) we get:

$$\mu(x) = A + e^{kx}\{c + a/b(x)\} \tag{A1.6}$$

which, after rewriting, gives (2.3).

The proof is now completed.

**A2. Derivation of (2.4)**

Consider first the frailty distribution at $x = x_0$. Let Z be the frailty variable at that age. According to (A1.5) Z has a generalized gamma distribution with parameters a, $b(x_0)$ and c. Then $U = Z - c$ is gamma distributed with parameters a and $b(x_0)$ . The mean and relative standard deviation of U is denoted by $\eta$ and $\alpha$ respectively, i.e.

$$\eta = a/b(x_0) \text{ and } \alpha^2 = 1/a \tag{A2.1a,b}$$

From (A1.4) we get for $x > x_0$: $b(x) - b(x_0) = (e^{ks} - e^{kx_0})/k =$

$= \int_{x_0}^{x} e^{kt}dt$ and thus $b(x) = b(x_0) + \int_{x_0}^{x} e^{kt}dt$ .

Inserting this expression for b(x) in (A1.6) and using (A2.1a,b) we get in the case of A = 0 the equation (2.4).

## Appendix B

France 1920 - 1929

| Age x | Men D(x)[1] | Men S(x)[2] | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 80 | 54310 | 348599 | .1558 | 68808 | 522694 | .1316 |
| 81 | 49162 | 293248 | .1676 | 62889 | 447506 | .1405 |
| 82 | 44937 | 244665 | .1837 | 59781 | 382626 | .1562 |
| 83 | 39429 | 197717 | .1994 | 53816 | 318354 | .1690 |
| 84 | 33777 | 156875 | .2153 | 48092 | 260979 | .1843 |
| 85 | 28236 | 122524 | .2305 | 41652 | 210693 | .1977 |
| 86 | 22989 | 92812 | .2477 | 35646 | 166215 | .2145 |
| 87 | 18466 | 68626 | .2691 | 29735 | 128312 | .2317 |
| 88 | 14098 | 49272 | .2861 | 23902 | 96805 | .2469 |
| 89 | 10565 | 34733 | .3042 | 19003 | 71437 | .2660 |
| 90 | 7955 | 24284 | .3276 | 14839 | 52225 | .2841 |
| 91 | 5544 | 16293 | .3403 | 10893 | 36848 | .2956 |
| 92 | 3922 | 10770 | .3642 | 8246 | 25632 | .3217 |
| 93 | 2677 | 6813 | .3929 | 5779 | 17002 | .3399 |
| 94 | 1661 | 4143 | .4009 | 3852 | 10943 | .3520 |
| 95 | 1066 | 2541 | .4195 | 2614 | 6950 | .3761 |
| 96 | 625 | 1466 | .4263 | 1719 | 4315 | .3984 |
| 97 | 365 | 855 | .4269 | 1074 | 2616 | .4106 |
| 98 | 234 | 497 | .4708 | 618 | 1497 | .4128 |
| 99 | 152 | 287 | .5296 | 405 | 874 | .4634 |
| 100 | 91 | 140 | .6500 | 217 | 470 | .4617 |
| 101 | 35 | 54 | .6481 | 144 | 259 | .5560 |
| 102 | 9 | 17 | .5294 | 71 | 113 | .6283 |
| 103 | 7 | 8 | .8750 | 30 | 40 | .7500 |
| 104 | 3 | 3 | 1.0000 | 11 | 11 | 1.0000 |

1) Number of deceased who should have attained age x+1 during 1921-29

2) Number of persons who attained age x during 1920-28

Source: Depoid,F.(1973): La mortalité des grands vieillards, Population 28, p 755-792

Swedish data 1967-73, 1973-79 and 1979-84 are collected from Population Changes (SOS), part 3, Statistics Sweden.

France 1929 - 1938

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 54388 | 371221 | .1465 | 69537 | 580480 | .1198 |
| 81 | 49951 | 312977 | .1596 | 65033 | 504689 | .1289 |
| 82 | 45132 | 256733 | .1758 | 61368 | 429139 | .1430 |
| 83 | 39384 | 209467 | .1880 | 56116 | 361555 | .1552 |
| 84 | 34057 | 169085 | .2014 | 50963 | 301297 | .1691 |
| 85 | 29054 | 132607 | .2191 | 44810 | 245138 | .1828 |
| 86 | 23989 | 101670 | .2360 | 38624 | 194881 | .1982 |
| 87 | 19150 | 75902 | .2523 | 32756 | 151474 | .2162 |
| 88 | 14795 | 55444 | .2668 | 26371 | 114302 | .2307 |
| 89 | 11374 | 40002 | .2843 | 21429 | 85286 | .2513 |
| 90 | 8460 | 27882 | .3034 | 16501 | 61730 | .2673 |
| 91 | 6034 | 19006 | .3175 | 12371 | 44061 | .2808 |
| 92 | 4200 | 12500 | .3360 | 9397 | 30740 | .3057 |
| 93 | 2904 | 7945 | .3655 | 6668 | 20507 | .3252 |
| 94 | 1727 | 4816 | .3586 | 4588 | 13397 | .3425 |
| 95 | 1143 | 2909 | .3929 | 3022 | 8486 | .3561 |
| 96 | 662 | 1668 | .3969 | 1993 | 5200 | .3833 |
| 97 | 426 | 917 | .4646 | 1299 | 3041 | .4272 |
| 98 | 212 | 448 | .4732 | 813 | 1606 | .5062 |
| 99 | 132 | 228 | .5789 | 397 | 777 | .5109 |
| 100 | 54 | 88 | .6136 | 186 | 365 | .5096 |
| 101 | 12 | 30 | .4000 | 86 | 166 | .5181 |
| 102 | 10 | 18 | .5556 | 46 | 77 | .5974 |
| 103 | 8 | 9 | .8889 | 24 | 34 | .7059 |
| 104 | 1 | 1 | 1.0000 | 8 | 8 | 1.0000 |

France 1948 - 1969

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 85 | 92961 | 482306 | .1927 | 155205 | 1002730 | .1548 |
| 86 | 80421 | 383714 | .2096 | 140784 | 829232 | .1698 |
| 87 | 67045 | 297910 | .2251 | 124192 | 670993 | .1851 |
| 88 | 55177 | 226308 | .2438 | 106574 | 532418 | .2002 |
| 89 | 43672 | 167893 | .2601 | 89461 | 414469 | .2158 |
| 90 | 33307 | 120636 | .2761 | 73645 | 313947 | .2346 |
| 91 | 25199 | 84580 | .2979 | 57988 | 231470 | .2505 |
| 92 | 18195 | 57320 | .3174 | 44788 | 166478 | .2690 |
| 93 | 12358 | 37508 | .3295 | 33281 | 116191 | .2864 |
| 94 | 8420 | 24063 | .3499 | 24071 | 79046 | .3045 |
| 95 | 5486 | 14961 | .3667 | 17025 | 52351 | .3252 |
| 96 | 3342 | 9095 | .3675 | 11571 | 33647 | .3439 |
| 97 | 2075 | 5495 | .3776 | 7629 | 20826 | .3663 |
| 98 | 1290 | 3301 | .3908 | 4686 | 12652 | .3704 |
| 99 | 936 | 2015 | .4645 | 3240 | 7955 | .4073 |
| 100 | 538 | 1069 | .5033 | 1973 | 4491 | .4393 |
| 101 | 226 | 484 | .4669 | 1054 | 2363 | .4460 |
| 102 | 113 | 243 | .4650 | 573 | 1227 | .4670 |
| 103 | 57 | 121 | .4711 | 298 | 611 | .4877 |
| 104 | 31 | 59 | .5254 | 147 | 291 | .5052 |
| 105 | 16 | 26 | .6154 | 70 | 134 | .5224 |
| 106 | 6 | 8 | .7500 | 33 | 61 | .5410 |
| 107 | 2 | 2 | 1.0000 | 17 | 26 | .6538 |
| 108 | 0 | 0 | - | 8 | 9 | .8889 |
| 109 | 0 | 0 | - | 1 | 1 | 1.0000 |

The Netherlands 1910 - 1925

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 9287 | 70407 | .1319 | 10387 | 85125 | .1220 |
| 81 | 8514 | 59950 | .1420 | 9751 | 73516 | .1326 |
| 82 | 7714 | 50395 | .1531 | 9077 | 62523 | .1452 |
| 83 | 6972 | 41634 | .1675 | 8179 | 52404 | .1561 |
| 84 | 6145 | 33815 | .1817 | 7373 | 43410 | .1698 |
| 85 | 5303 | 27051 | .1960 | 6495 | 35401 | .1835 |
| 86 | 4627 | 21320 | .2170 | 5566 | 28370 | .1962 |
| 87 | 3814 | 16291 | .2341 | 4847 | 22267 | .2177 |
| 88 | 3091 | 12274 | .2518 | 3947 | 17119 | .2306 |
| 89 | 2431 | 8984 | .2706 | 3138 | 12920 | .2429 |
| 90 | 1827 | 6369 | .2869 | 2547 | 9492 | .2683 |
| 91 | 1324 | 4446 | .2978 | 1877 | 6860 | .2736 |
| 92 | 978 | 3069 | .3187 | 1461 | 4867 | .3002 |
| 93 | 663 | 2050 | .3234 | 1074 | 3338 | .3218 |
| 94 | 473 | 1337 | .3538 | 749 | 2207 | .3394 |
| 95 | 337 | 856 | .3937 | 569 | 1438 | .3957 |
| 96 | 189 | 486 | .3889 | 311 | 856 | .3633 |
| 97 | 111 | 285 | .3895 | 216 | 538 | .4015 |
| 98 | 67 | 172 | .3895 | 122 | 320 | .3813 |
| 99 | 46 | 100 | .4600 | 73 | 185 | .3946 |
| 100 | 23 | 51 | .4510 | 62 | 112 | .5536 |
| 101 | 15 | 29 | .5172 | 20 | 46 | .4348 |
| 102 | 9 | 13 | .6923 | 9 | 22 | .4091 |
| 103 | 2 | 4 | .5000 | 9 | 11 | .8182 |
| 104 | 2 | 2 | 1.0000 | 3 | 3 | 1.0000 |

The Netherlands 1925 - 1945

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 16758 | 128790 | .1301 | 17633 | 146315 | .1205 |
| 81 | 15509 | 109673 | .1414 | 16621 | 126249 | .1317 |
| 82 | 14105 | 92136 | .1531 | 15238 | 107491 | .1418 |
| 83 | 12878 | 76862 | .1675 | 14183 | 91021 | .1558 |
| 84 | 11563 | 62982 | .1836 | 12830 | 75580 | .1698 |
| 85 | 10029 | 50864 | .1972 | 11648 | 62282 | .1870 |
| 86 | 8758 | 40308 | .2173 | 9979 | 50286 | .1984 |
| 87 | 7198 | 31327 | .2298 | 8754 | 40262 | .2174 |
| 88 | 5950 | 23916 | .2488 | 7393 | 31318 | .2361 |
| 89 | 4717 | 17840 | .2644 | 5939 | 23794 | .2496 |
| 90 | 3795 | 13050 | .2908 | 4849 | 17846 | .2717 |
| 91 | 2878 | 9151 | .3145 | 3738 | 12910 | .2895 |
| 92 | 2011 | 6212 | .3237 | 2859 | 9166 | .3119 |
| 93 | 1455 | 4164 | .3494 | 2090 | 6236 | .3352 |
| 94 | 1022 | 2697 | .3789 | 1487 | 4141 | .3591 |
| 95 | 645 | 1645 | .3921 | 948 | 2649 | .3579 |
| 96 | 410 | 1001 | .4096 | 690 | 1679 | .4110 |
| 97 | 246 | 590 | .4169 | 426 | 1000 | .4260 |
| 98 | 151 | 344 | .4390 | 258 | 580 | .4448 |
| 99 | 80 | 189 | .4233 | 179 | 335 | .5343 |
| 100 | 57 | 109 | .5229 | 78 | 161 | .4845 |
| 101 | 35 | 51 | .6863 | 44 | 87 | .5057 |
| 102 | 9 | 17 | .5294 | 31 | 46 | .6739 |
| 103 | 7 | 8 | .8750 | 11 | 17 | .6471 |
| 104 | 0 | 0 | - | 3 | 5 | .6000 |
| 105 | 0 | 0 | - | 1 | 2 | .5000 |
| 106 | 0 | 0 | - | 1 | 1 | 1.0000 |

The Netherlands 1945 - 1970

| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 85 | 21709 | 132099 | .1643 | 24568 | 159853 | .1537 |
| 86 | 19438 | 106631 | .1823 | 21976 | 130530 | .1684 |
| 87 | 16464 | 84134 | .1957 | 19136 | 104529 | .1831 |
| 88 | 13978 | 65006 | .2150 | 16134 | 82099 | .1965 |
| 89 | 11291 | 48826 | .2313 | 13313 | 63361 | .2101 |
| 90 | 8856 | 35930 | .2465 | 10906 | 47833 | .2280 |
| 91 | 6846 | 25860 | .2647 | 8603 | 35196 | .2444 |
| 92 | 5224 | 18158 | .2877 | 6755 | 25398 | .2660 |
| 93 | 3763 | 12274 | .3066 | 4921 | 17706 | .2779 |
| 94 | 2641 | 8062 | .3276 | 3549 | 12082 | .2937 |
| 95 | 1799 | 5139 | .3501 | 2579 | 8063 | .3199 |
| 96 | 1122 | 3167 | .3543 | 1737 | 5179 | .3354 |
| 97 | 762 | 1931 | .3946 | 1117 | 3239 | .3449 |
| 98 | 433 | 1099 | .3940 | 771 | 1983 | .3888 |
| 99 | 256 | 631 | .4057 | 432 | 1135 | .3806 |
| 100 | 169 | 352 | .4801 | 277 | 653 | .4242 |
| 101 | 86 | 173 | .4971 | 150 | 352 | .4261 |
| 102 | 43 | 81 | .5309 | 79 | 188 | .4202 |
| 103 | 16 | 35 | .4571 | 45 | 100 | .4500 |
| 104 | 11 | 19 | .5789 | 19 | 52 | .3654 |
| 105 | 2 | 7 | .2857 | 12 | 31 | .3871 |
| 106 | 3 | 5 | .6000 | 7 | 18 | .3889 |
| 107 | 1 | 2 | .5000 | 6 | 10 | .6000 |
| 108 | 0 | 1 | .0000 | 2 | 4 | .5000 |
| 109 | 1 | 1 | 1.0000 | 1 | 2 | .5000 |
| 110 | 0 | 0 | - | 1 | 1 | 1.0000 |

Switzerland 1876 - 1914

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 11573 | 67933 | .1704 | 12910 | 79964 | .1614 |
| 81 | 10142 | 55715 | .1820 | 11311 | 65725 | .1721 |
| 82 | 8706 | 45059 | .1932 | 9856 | 53352 | .1847 |
| 83 | 7360 | 35883 | .2051 | 8429 | 42529 | .1982 |
| 84 | 6189 | 28185 | .2196 | 7101 | 33326 | .2131 |
| 85 | 5113 | 21708 | .2355 | 5848 | 25758 | .2270 |
| 86 | 4157 | 16427 | .2531 | 4753 | 19479 | .2440 |
| 87 | 3273 | 12055 | .2715 | 3666 | 14392 | .2547 |
| 88 | 2516 | 8678 | .2899 | 2832 | 10452 | .2710 |
| 89 | 1916 | 6072 | .3155 | 2090 | 7422 | .2816 |
| 90 | 1349 | 4099 | .3291 | 1642 | 5180 | .3170 |
| 91 | 931 | 2715 | .3429 | 1137 | 3454 | .3292 |
| 92 | 631 | 1753 | .3600 | 756 | 2260 | .3345 |
| 93 | 409 | 1117 | .3662 | 518 | 1455 | .3560 |
| 94 | 270 | 694 | .3890 | 360 | 911 | .3952 |
| 95 | 176 | 412 | .4272 | 209 | 536 | .3899 |
| 96 | 117 | 238 | .4916 | 130 | 322 | .4037 |
| 97 | 55 | 118 | .4661 | 99 | 196 | .5051 |
| 98 | 27 | 64 | .4219 | 33 | 98 | .3367 |
| 99 | 13 | 36 | .3611 | 30 | 63 | .4762 |
| 100 | 10 | 23 | .4348 | 15 | 32 | .4688 |
| 101 | 6 | 12 | .5000 | 9 | 15 | .6000 |
| 102 | 0 | 6 | .0000 | 3 | 5 | .6000 |
| 103 | 5 | 5 | 1.0000 | 0 | 2 | .0000 |
| 104 | 0 | 0 | - | 1 | 2 | .5000 |
| 105 | 0 | 0 | - | 0 | 1 | .0000 |
| 106 | 0 | 0 | - | 1 | 1 | 1.0000 |

Switzerland 1914 - 1948

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 13797 | 94451 | .1461 | 17928 | 140467 | .1276 |
| 81 | 12582 | 78729 | .1598 | 16563 | 119498 | .1386 |
| 82 | 11053 | 64433 | .1715 | 15290 | 99994 | .1529 |
| 83 | 9708 | 52239 | .1858 | 13787 | 82281 | .1676 |
| 84 | 8342 | 41490 | .2011 | 12298 | 66529 | .1849 |
| 85 | 7001 | 32318 | .2166 | 10011 | 52418 | .1910 |
| 86 | 5730 | 24667 | .2323 | 8680 | 41108 | .2112 |
| 87 | 4679 | 18500 | .2529 | 7187 | 31450 | .2285 |
| 88 | 3607 | 13507 | .2670 | 5664 | 23432 | .2417 |
| 89 | 2704 | 9667 | .2797 | 4477 | 17165 | .2608 |
| 90 | 2085 | 6810 | .3062 | 3344 | 12311 | .2716 |
| 91 | 1467 | 4608 | .3184 | 2606 | 8646 | .3014 |
| 92 | 1021 | 3065 | .3331 | 1866 | 5840 | .3195 |
| 93 | 721 | 1984 | .3634 | 1273 | 3847 | .3309 |
| 94 | 452 | 1224 | .3693 | 845 | 2517 | .3357 |
| 95 | 309 | 760 | .4066 | 599 | 1598 | .3748 |
| 96 | 179 | 427 | .4192 | 378 | 962 | .3929 |
| 97 | 93 | 238 | .3908 | 216 | 557 | .3878 |
| 98 | 63 | 139 | .4532 | 134 | 333 | .4024 |
| 99 | 36 | 76 | .4737 | 91 | 190 | .4789 |
| 100 | 19 | 37 | .5135 | 42 | 97 | .4330 |
| 101 | 7 | 19 | .3684 | 33 | 54 | .6111 |
| 102 | 5 | 12 | .4167 | 10 | 20 | .5000 |
| 103 | 5 | 7 | .7143 | 5 | 10 | .5000 |
| 104 | 0 | 2 | .0000 | 3 | 4 | .7500 |
| 105 | 2 | 2 | 1.0000 | 1 | 1 | 1.0000 |

Switzerland 1948 - 1970

| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 85 | 9594 | 51823 | .1851 | 14162 | 88268 | .1604 |
| 86 | 8376 | 40963 | .2045 | 12819 | 71921 | .1782 |
| 87 | 6912 | 31507 | .2194 | 10970 | 57184 | .1918 |
| 88 | 5647 | 23687 | .2384 | 9286 | 44723 | .2076 |
| 89 | 4508 | 17372 | .2595 | 7760 | 34120 | .2274 |
| 90 | 3346 | 12360 | .2707 | 6100 | 25328 | .2408 |
| 91 | 2540 | 8654 | .2935 | 4683 | 18432 | .2541 |
| 92 | 1782 | 5858 | .3042 | 3576 | 13101 | .2730 |
| 93 | 1238 | 3869 | .3200 | 2663 | 9047 | .2944 |
| 94 | 846 | 2503 | .3380 | 1924 | 6015 | .3199 |
| 95 | 587 | 1565 | .3751 | 1318 | 3877 | .3400 |
| 96 | 356 | 934 | .3812 | 852 | 2418 | .3524 |
| 97 | 220 | 547 | .4022 | 535 | 1487 | .3598 |
| 98 | 122 | 309 | .3948 | 365 | 888 | .4110 |
| 99 | 84 | 174 | .4828 | 199 | 495 | .4020 |
| 100 | 43 | 86 | .5000 | 129 | 277 | .4657 |
| 101 | 20 | 40 | .5000 | 62 | 141 | .4397 |
| 102 | 11 | 18 | .6111 | 31 | 75 | .4133 |
| 103 | 4 | 7 | .5714 | 22 | 40 | .5500 |
| 104 | 0 | 3 | .0000 | 12 | 17 | .7059 |
| 105 | 3 | 3 | 1.0000 | 2 | 4 | .5000 |
| 106 | 0 | 0 | - | 1 | 2 | .5000 |
| 107 | 0 | 0 | - | 0 | 1 | .0000 |
| 108 | 0 | 0 | - | 1 | 1 | 1.0000 |

Sweden 1901 - 1914

| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 80 | 9196 | 77650 | .1184 | 11040 | 101554 | .1087 |
| 81 | 8613 | 67019 | .1285 | 10564 | 88739 | .1190 |
| 82 | 8208 | 57273 | .1433 | 10082 | 76905 | .1311 |
| 83 | 7639 | 48379 | .1579 | 9523 | 65866 | .1446 |
| 84 | 6805 | 39641 | .1717 | 8691 | 54990 | .1580 |
| 85 | 6016 | 31905 | .1886 | 7747 | 45366 | .1708 |
| 86 | 5255 | 25208 | .2085 | 6826 | 36803 | .1855 |
| 87 | 4221 | 19289 | .2188 | 6014 | 29226 | .2058 |
| 88 | 3438 | 14295 | .2405 | 4852 | 22199 | .2186 |
| 89 | 2716 | 10343 | .2626 | 3971 | 16611 | .2391 |
| 90 | 2067 | 7271 | .2843 | 3107 | 12222 | .2542 |
| 91 | 1466 | 4894 | .2996 | 2351 | 8683 | .2708 |
| 92 | 1008 | 3139 | .3211 | 1762 | 5933 | .2970 |
| 93 | 643 | 1964 | .3274 | 1261 | 3975 | .3172 |
| 94 | 442 | 1240 | .3565 | 852 | 2614 | .3259 |
| 95 | 311 | 734 | .4237 | 596 | 1686 | .3535 |
| 96 | 167 | 390 | .4282 | 418 | 1029 | .4062 |
| 97 | 95 | 213 | .4460 | 227 | 581 | .3907 |
| 98 | 41 | 113 | .3628 | 136 | 338 | .4024 |
| 99 | 33 | 69 | .4783 | 76 | 191 | .3979 |
| 100 | 21 | 31 | .6774 | 48 | 112 | .4286 |
| 101 | 3 | 9 | .3333 | 33 | 60 | .5500 |
| 102 | 4 | 7 | .5714 | 11 | 26 | .4231 |
| 103 | 1 | 3 | .3333 | 10 | 16 | .6250 |
| 104 | 1 | 2 | .5000 | 1 | 5 | .2000 |
| 105 | 1 | 1 | 1.0000 | 1 | 4 | .2500 |
| 106 | 0 | 0 | - | 2 | 2 | 1.0000 |

Sweden 1914 - 1930

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 12008 | 104329 | .1151 | 14450 | 136788 | .1056 |
| 81 | 11682 | 91664 | .1274 | 14434 | 121423 | .1189 |
| 82 | 11142 | 79555 | .1401 | 13819 | 106215 | .1301 |
| 83 | 10202 | 67757 | .1506 | 12913 | 91768 | .1407 |
| 84 | 9748 | 57364 | .1699 | 12252 | 78912 | .1553 |
| 85 | 8690 | 47341 | .1836 | 11401 | 66217 | .1722 |
| 86 | 7558 | 38069 | .1985 | 10025 | 54232 | .1849 |
| 87 | 6557 | 30146 | .2175 | 8825 | 43732 | .2018 |
| 88 | 5591 | 23455 | .2384 | 7470 | 34710 | .2152 |
| 89 | 4543 | 17731 | .2562 | 6361 | 27154 | .2343 |
| 90 | 3634 | 13033 | .2788 | 5243 | 20608 | .2544 |
| 91 | 2752 | 9337 | .2947 | 4086 | 15333 | .2665 |
| 92 | 2080 | 6580 | .3161 | 3232 | 11235 | .2877 |
| 93 | 1469 | 4457 | .3296 | 2376 | 7890 | .3011 |
| 94 | 1051 | 2916 | .3604 | 1757 | 5392 | .3259 |
| 95 | 668 | 1805 | .3701 | 1286 | 3536 | .3637 |
| 96 | 421 | 1091 | .3859 | 845 | 2207 | .3829 |
| 97 | 287 | 646 | .4443 | 511 | 1312 | .3895 |
| 98 | 156 | 346 | .4509 | 320 | 781 | .4097 |
| 99 | 68 | 176 | .3864 | 172 | 447 | .3848 |
| 100 | 48 | 102 | .4706 | 110 | 246 | .4472 |
| 101 | 27 | 51 | .5294 | 60 | 127 | .4724 |
| 102 | 10 | 20 | .5000 | 36 | 66 | .5455 |
| 103 | 2 | 9 | .2222 | 17 | 26 | .6538 |
| 104 | 2 | 4 | .5000 | 7 | 10 | .7000 |
| 105 | 2 | 2 | 1.0000 | 2 | 2 | 1.0000 |
| 106 | 0 | 0 | - | 1 | 1 | 1.0000 |

Sweden 1930 - 1945

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 80 | 13554 | 117280 | .1156 | 15979 | 148609 | .1075 |
| 81 | 12995 | 102443 | .1269 | 15582 | 131156 | .1188 |
| 82 | 12250 | 87797 | .1395 | 14918 | 113868 | .1310 |
| 83 | 11260 | 74244 | .1517 | 14039 | 97141 | .1445 |
| 84 | 10296 | 62077 | .1659 | 12763 | 81731 | .1562 |
| 85 | 9209 | 50818 | .1812 | 11674 | 67944 | .1718 |
| 86 | 8136 | 40957 | .1986 | 10392 | 55497 | .1873 |
| 87 | 7054 | 32236 | .2188 | 9057 | 44495 | .2036 |
| 88 | 5722 | 24906 | .2297 | 7705 | 35149 | .2192 |
| 89 | 4756 | 19017 | .2501 | 6441 | 27250 | .2364 |
| 90 | 3870 | 14130 | .2739 | 5239 | 20659 | .2536 |
| 91 | 3008 | 10058 | .2991 | 4210 | 15142 | .2780 |
| 92 | 2129 | 6909 | .3081 | 3199 | 10764 | .2972 |
| 93 | 1522 | 4711 | .3231 | 2422 | 7588 | .3192 |
| 94 | 1135 | 3139 | .3616 | 1694 | 5126 | .3305 |
| 95 | 765 | 2015 | .3797 | 1199 | 3456 | .3469 |
| 96 | 493 | 1260 | .3913 | 853 | 2226 | .3832 |
| 97 | 315 | 759 | .4150 | 516 | 1380 | .3739 |
| 98 | 197 | 434 | .4539 | 364 | 865 | .4208 |
| 99 | 104 | 239 | .4351 | 226 | 507 | .4458 |
| 100 | 74 | 138 | .5362 | 137 | 292 | .4692 |
| 101 | 32 | 65 | .4923 | 78 | 155 | .5032 |
| 102 | 24 | 37 | .6486 | 41 | 78 | .5256 |
| 103 | 8 | 13 | .6154 | 24 | 41 | .5854 |
| 104 | 3 | 8 | .3750 | 7 | 17 | .4118 |
| 105 | 3 | 5 | .6000 | 7 | 8 | .8750 |
| 106 | 2 | 2 | 1.0000 | 1 | 1 | 1.0000 |

Sweden 1945 - 1967

| Age | Men | | | Women | | |
|---|---|---|---|---|---|---|
| x | D(x) | S(x) | q(x) | D(x) | S(x) | q(x) |
| 85 | 17593 | 101864 | .1727 | 20787 | 129218 | .1609 |
| 86 | 15767 | 83070 | .1898 | 19078 | 107416 | .1776 |
| 87 | 13702 | 66258 | .2068 | 16853 | 87275 | .1931 |
| 88 | 11579 | 51354 | .2255 | 14280 | 69156 | .2065 |
| 89 | 9441 | 38973 | .2422 | 12145 | 53874 | .2254 |
| 90 | 7473 | 28815 | .2593 | 9858 | 40896 | .2411 |
| 91 | 5899 | 20954 | .2815 | 7886 | 30652 | .2573 |
| 92 | 4441 | 14773 | .3006 | 6252 | 22373 | .2794 |
| 93 | 3257 | 10124 | .3217 | 4729 | 15784 | .2996 |
| 94 | 2341 | 6742 | .3472 | 3403 | 10890 | .3125 |
| 95 | 1606 | 4306 | .3730 | 2494 | 7326 | .3404 |
| 96 | 1024 | 2636 | .3885 | 1724 | 4761 | .3621 |
| 97 | 650 | 1581 | .4111 | 1130 | 2989 | .3781 |
| 98 | 374 | 916 | .4083 | 692 | 1826 | .3790 |
| 99 | 236 | 534 | .4419 | 446 | 1109 | .4022 |
| 100 | 126 | 292 | .4315 | 286 | 654 | .4373 |
| 101 | 77 | 160 | .4813 | 153 | 360 | .4250 |
| 102 | 41 | 77 | .5325 | 95 | 198 | .4798 |
| 103 | 23 | 34 | .6765 | 48 | 98 | .4898 |
| 104 | 7 | 10 | .7000 | 29 | 46 | .6304 |
| 105 | 2 | 3 | .6667 | 7 | 18 | .3889 |
| 106 | 1 | 1 | 1.0000 | 4 | 10 | .4000 |
| 107 | 0 | 0 | - | 4 | 5 | .8000 |
| 108 | 0 | 0 | - | 0 | 1 | .0000 |
| 109 | 0 | 0 | - | 1 | 1 | 1.0000 |

Sweden 1967 - 1973

| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 85 | 6160 | 39504 | .1559 | 7462 | 57193 | .1305 |
| 86 | 5516 | 32209 | .1713 | 6809 | 47207 | .1442 |
| 87 | 4777 | 25794 | .1852 | 5896 | 38363 | .1537 |
| 88 | 4074 | 20428 | .1994 | 5288 | 31019 | .1705 |
| 89 | 3429 | 15755 | .2176 | 4422 | 24161 | .1830 |
| 90 | 2792 | 12097 | .2308 | 3823 | 18727 | .2041 |
| 91 | 2262 | 9089 | .2489 | 2997 | 14206 | .2110 |
| 92 | 1770 | 6625 | .2672 | 2552 | 10656 | .2395 |
| 93 | 1320 | 4670 | .2827 | 1867 | 7618 | .2451 |
| 94 | 928 | 3131 | .2964 | 1444 | 5305 | .2722 |
| 95 | 657 | 2067 | .3179 | 1034 | 3579 | .2889 |
| 96 | 449 | 1279 | .3511 | 752 | 2416 | .3113 |
| 97 | 271 | 761 | .3561 | 514 | 1546 | .3325 |
| 98 | 173 | 458 | .3777 | 321 | 963 | .3333 |
| 99 | 106 | 253 | .4190 | 209 | 566 | .3693 |
| 100 | 59 | 133 | .4436 | 136 | 319 | .4263 |
| 101 | 31 | 69 | .4493 | 72 | 173 | .4162 |
| 102 | 14 | 34 | .4118 | 39 | 98 | .3980 |
| 103 | 7 | 17 | .4118 | 26 | 55 | .4727 |
| 104 | 3 | 8 | .3750 | 17 | 28 | .6071 |
| 105 | 5 | 7 | .7143 | 8 | 15 | .5333 |
| 106 | 1 | 1 | 1.0000 | 7 | 8 | .8750 |
| 107 | 0 | 0 | - | 1 | 1 | 1.0000 |

Sweden 1973 - 1979

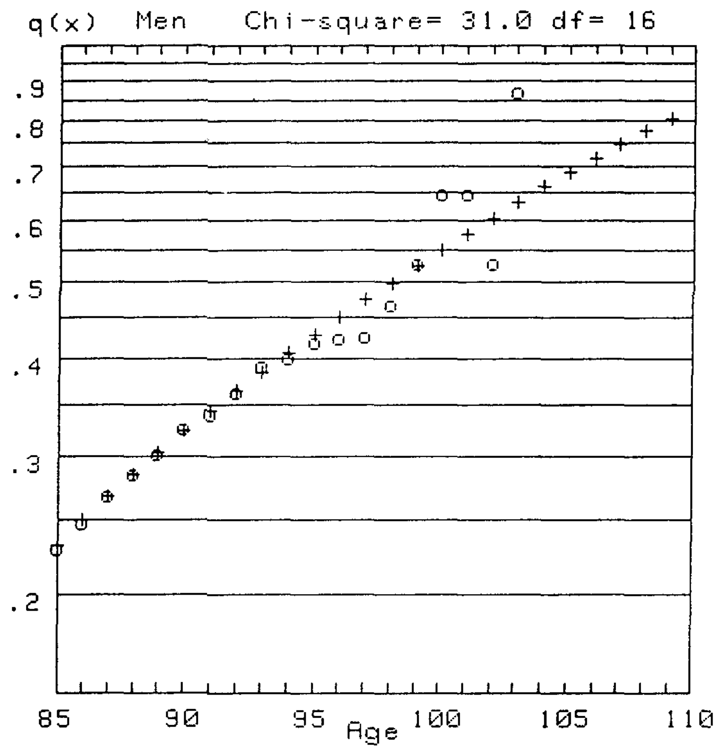
| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 85 | 6752 | 43313 | .1559 | 8686 | 72311 | .1201 |
| 86 | 6129 | 36045 | .1700 | 8153 | 61288 | .1330 |
| 87 | 5502 | 29772 | .1848 | 7398 | 51147 | .1446 |
| 88 | 4813 | 23926 | .2012 | 6596 | 41729 | .1581 |
| 89 | 3966 | 18979 | .2090 | 5759 | 33723 | .1708 |
| 90 | 3377 | 14775 | .2286 | 5036 | 26731 | .1884 |
| 91 | 2703 | 11099 | .2435 | 4154 | 20491 | .2027 |
| 92 | 2138 | 8076 | .2647 | 3262 | 15235 | .2141 |
| 93 | 1627 | 5716 | .2846 | 2735 | 11208 | .2440 |
| 94 | 1186 | 3952 | .3001 | 2036 | 7996 | .2546 |
| 95 | 895 | 2640 | .3390 | 1490 | 5462 | .2728 |
| 96 | 574 | 1694 | .3388 | 1050 | 3650 | .2877 |
| 97 | 389 | 1106 | .3517 | 768 | 2404 | .3195 |
| 98 | 258 | 703 | .3670 | 544 | 1549 | .3512 |
| 99 | 159 | 436 | .3647 | 320 | 922 | .3471 |
| 100 | 99 | 244 | .4057 | 203 | 557 | .3645 |
| 101 | 55 | 125 | .4400 | 131 | 327 | .4006 |
| 102 | 30 | 59 | .5085 | 66 | 168 | .3929 |
| 103 | 10 | 27 | .3704 | 43 | 92 | .4674 |
| 104 | 11 | 18 | .6111 | 24 | 44 | .5455 |
| 105 | 3 | 5 | .6000 | 6 | 15 | .4000 |
| 106 | 1 | 3 | .3333 | 6 | 7 | .8571 |
| 107 | 2 | 2 | 1.0000 | 1 | 1 | 1.0000 |

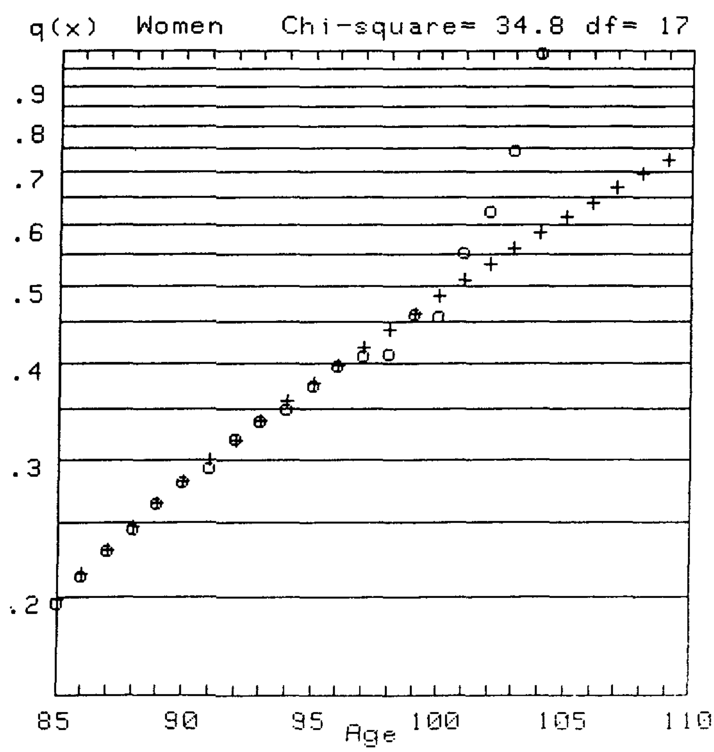
Sweden 1979 - 1984

| Age x | Men D(x) | Men S(x) | Men q(x) | Women D(x) | Women S(x) | Women q(x) |
|---|---|---|---|---|---|---|
| 85 | 6145 | 41137 | .1494 | 8347 | 73627 | .1134 |
| 86 | 5475 | 34061 | .1607 | 7441 | 62924 | .1183 |
| 87 | 4882 | 27726 | .1761 | 7125 | 53400 | .1334 |
| 88 | 4204 | 22058 | .1906 | 6643 | 44492 | .1493 |
| 89 | 3642 | 17347 | .2100 | 5942 | 36056 | .1648 |
| 90 | 2925 | 13320 | .2196 | 5071 | 28605 | .1773 |
| 91 | 2439 | 10117 | .2411 | 4307 | 22266 | .1934 |
| 92 | 1972 | 7594 | .2597 | 3679 | 17170 | .2143 |
| 93 | 1572 | 5529 | .2843 | 2960 | 12821 | .2309 |
| 94 | 1143 | 3917 | .2918 | 2324 | 9334 | .2490 |
| 95 | 856 | 2717 | .3151 | 1771 | 6693 | .2646 |
| 96 | 601 | 1857 | .3236 | 1370 | 4684 | .2925 |
| 97 | 413 | 1171 | .3527 | 927 | 3071 | .3019 |
| 98 | 259 | 704 | .3679 | 641 | 1964 | .3264 |
| 99 | 155 | 394 | .3934 | 393 | 1216 | .3232 |
| 100 | 96 | 243 | .3951 | 256 | 733 | .3493 |
| 101 | 82 | 146 | .5616 | 169 | 433 | .3903 |
| 102 | 29 | 71 | .4085 | 91 | 238 | .3824 |
| 103 | 22 | 40 | .5500 | 65 | 146 | .4452 |
| 104 | 9 | 17 | .5294 | 30 | 74 | .4054 |
| 105 | 5 | 7 | .7143 | 26 | 40 | .6500 |
| 106 | 1 | 1 | 1.0000 | 5 | 12 | .4167 |
| 107 | 0 | 0 | - | 4 | 6 | .6667 |

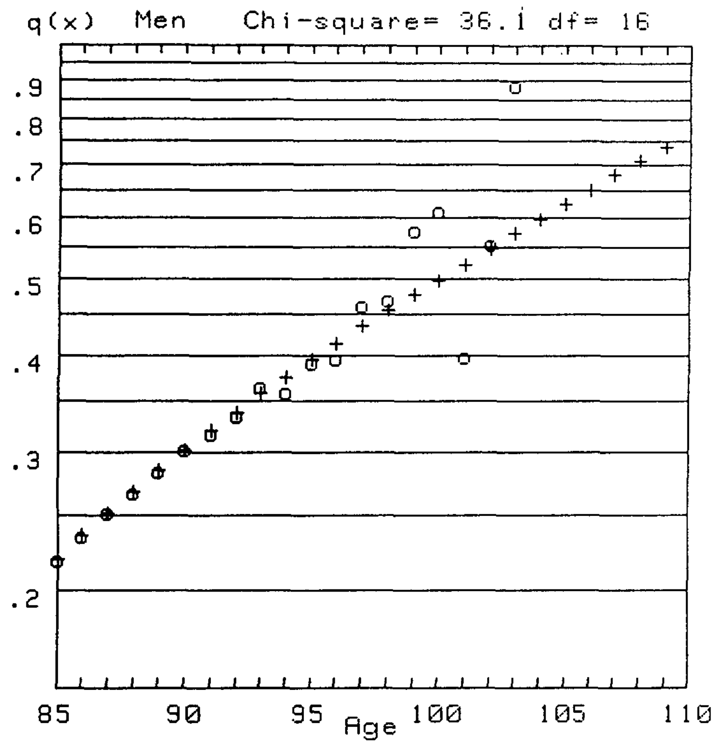
# Appendix C

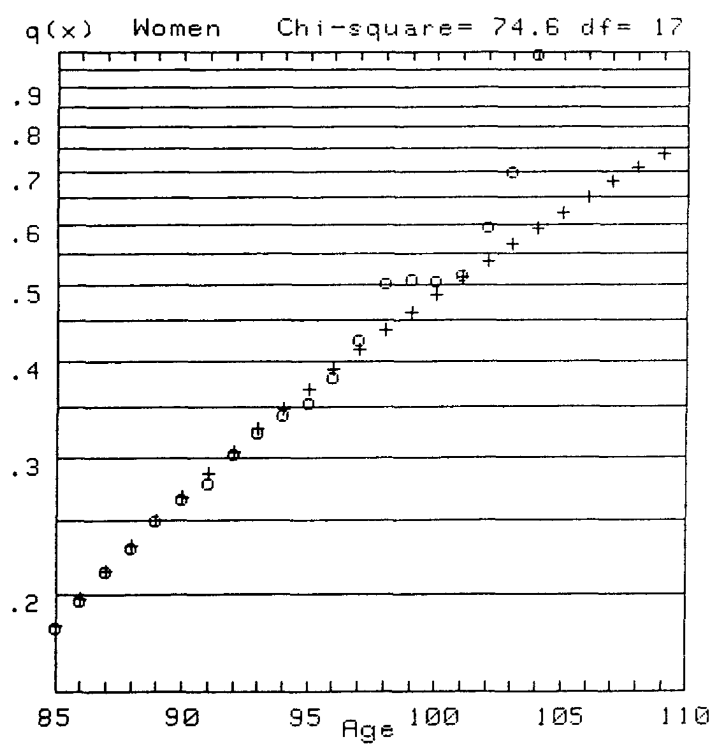
## France 1920-1929



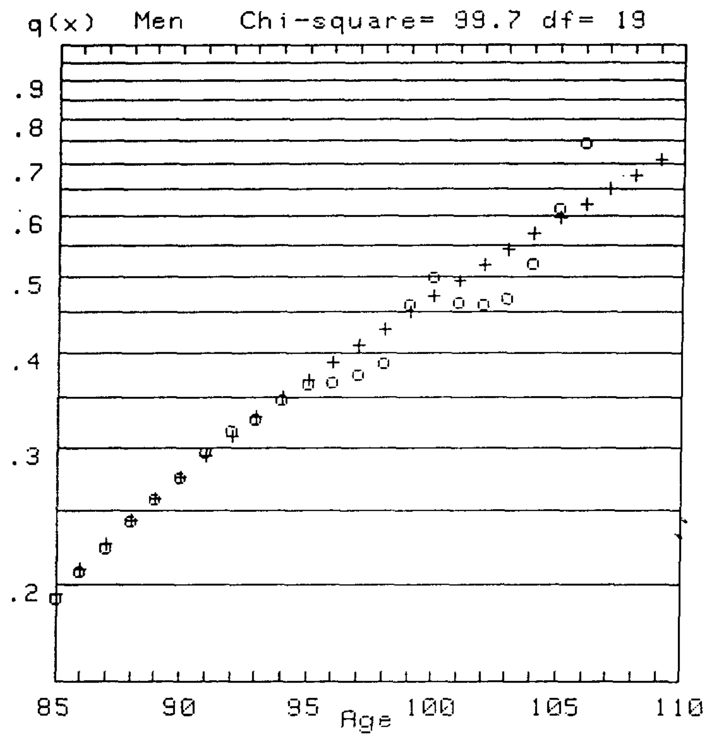



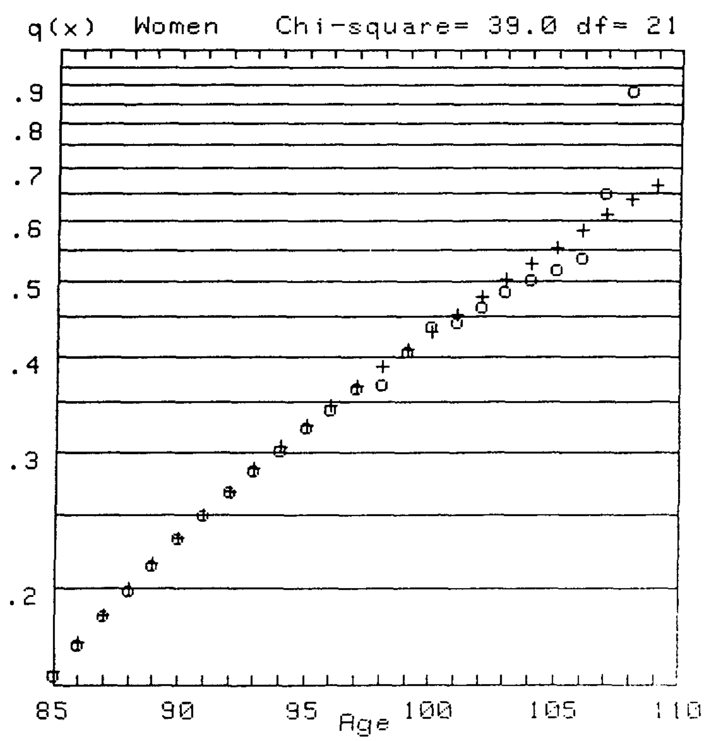
France 1929-1938



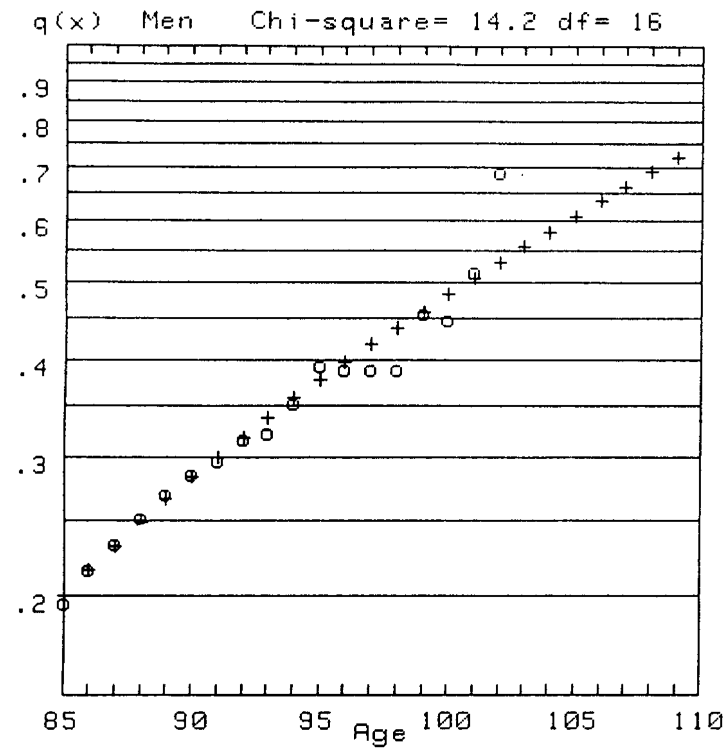



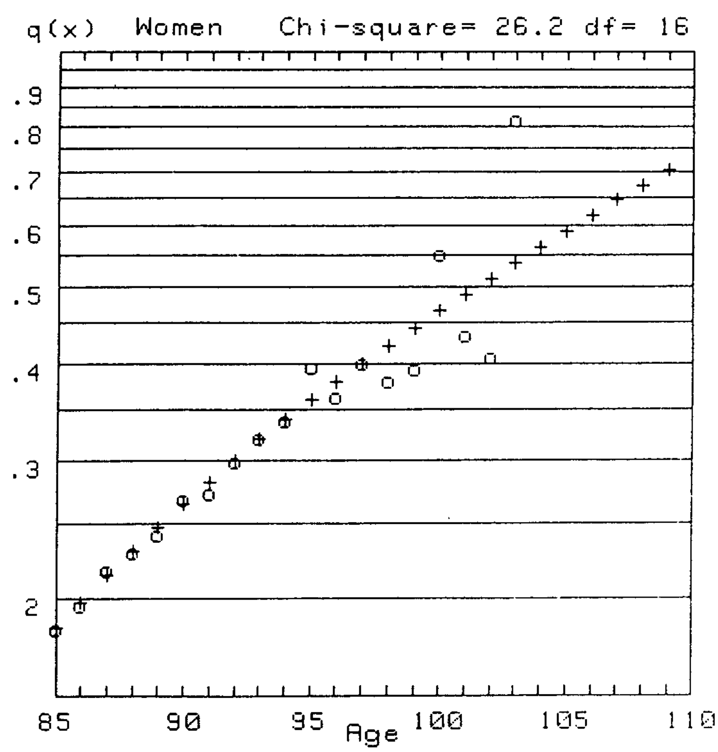
France 1948-1969



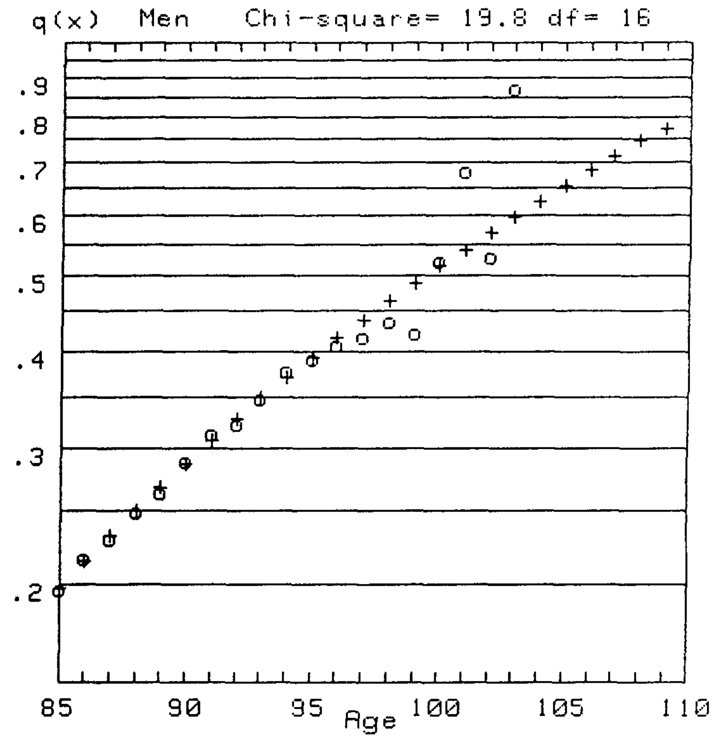



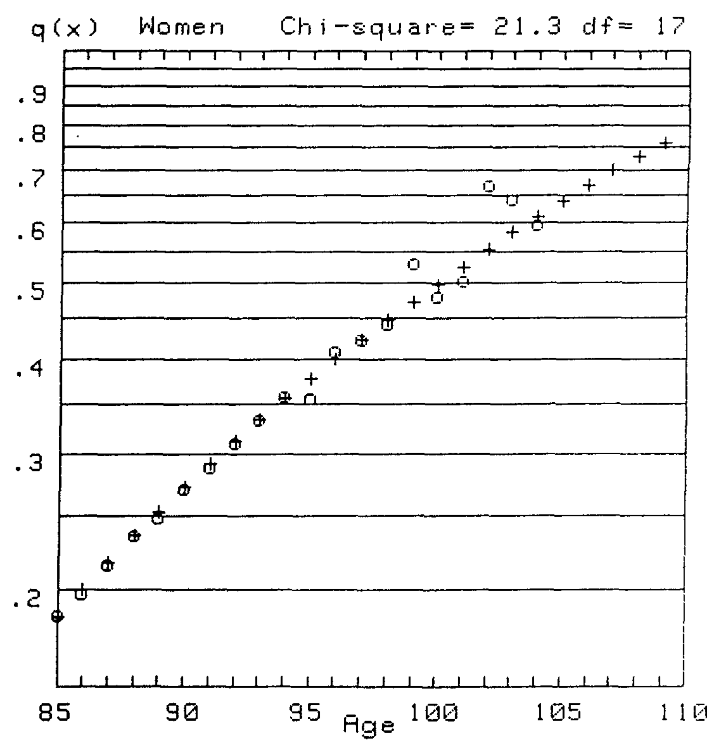
The Netherlands 1910-1925



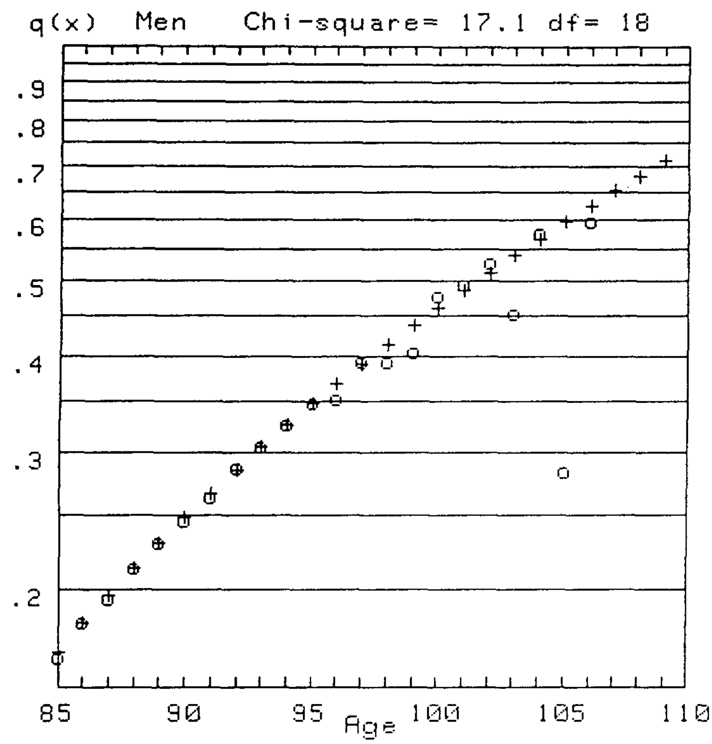



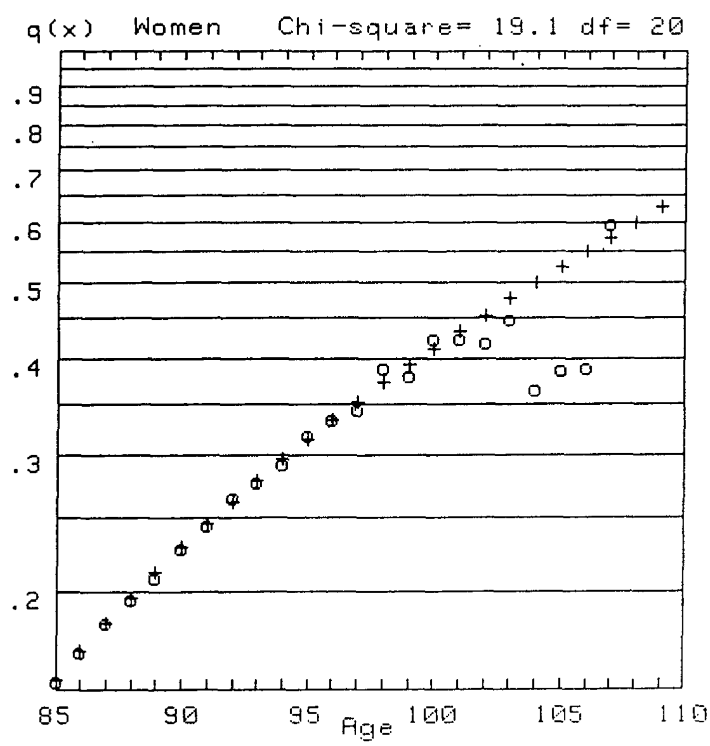
The Netherlands 1925-1945



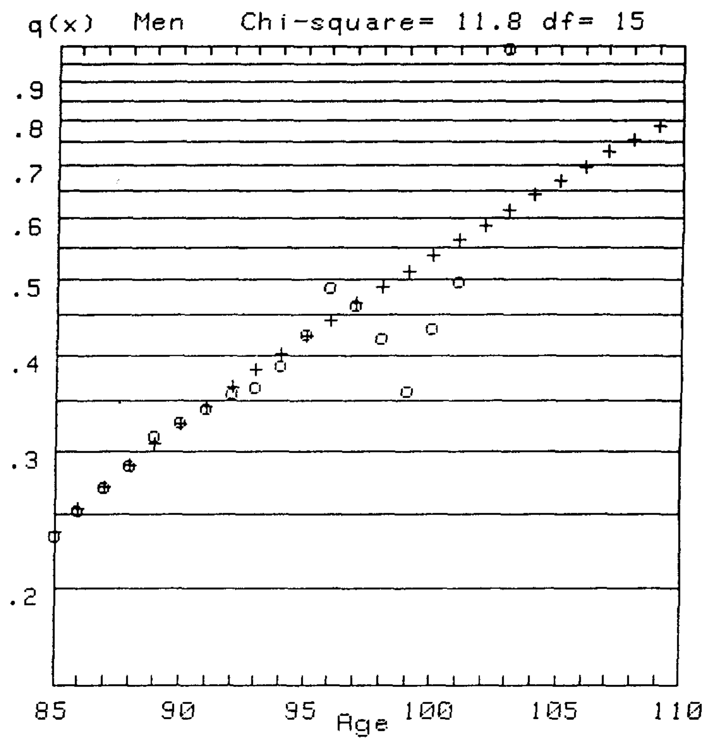



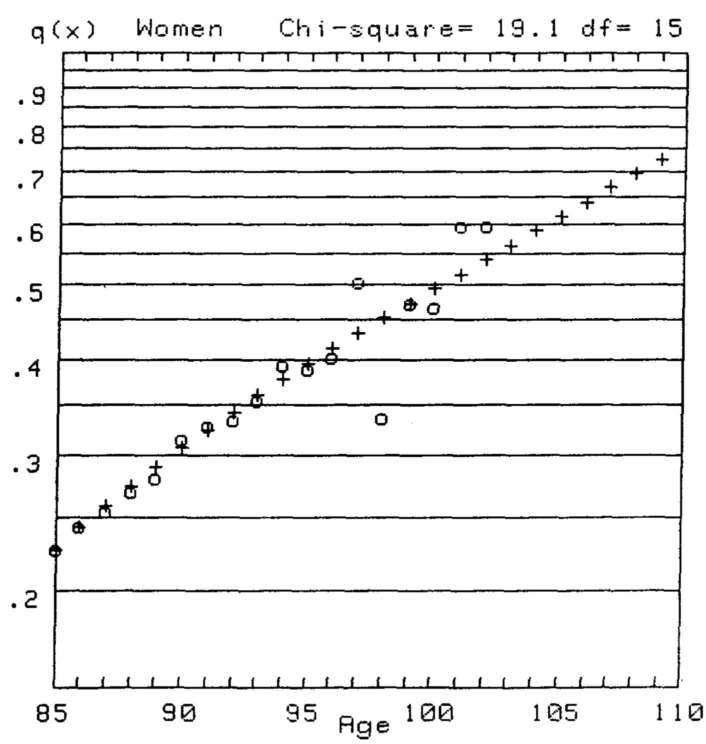
The Netherlands 1945-1970



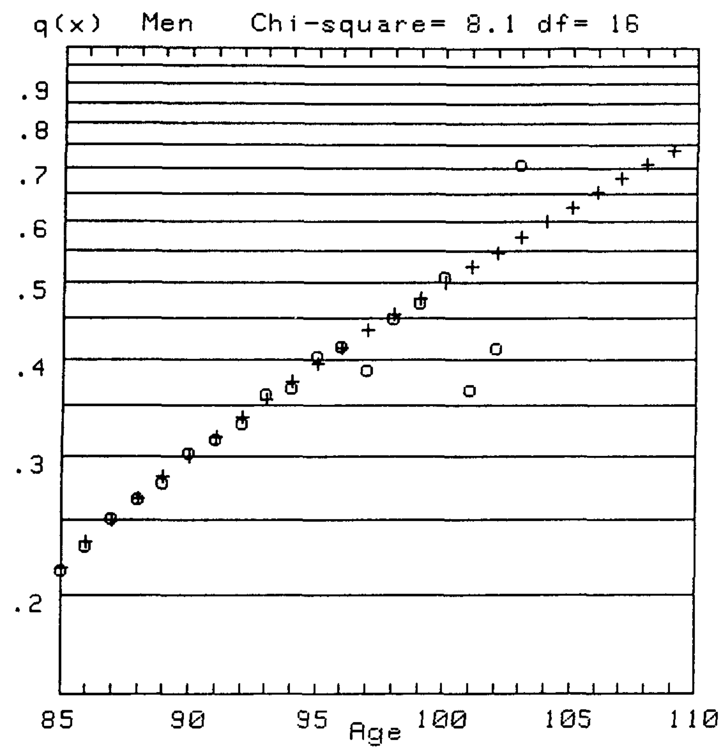



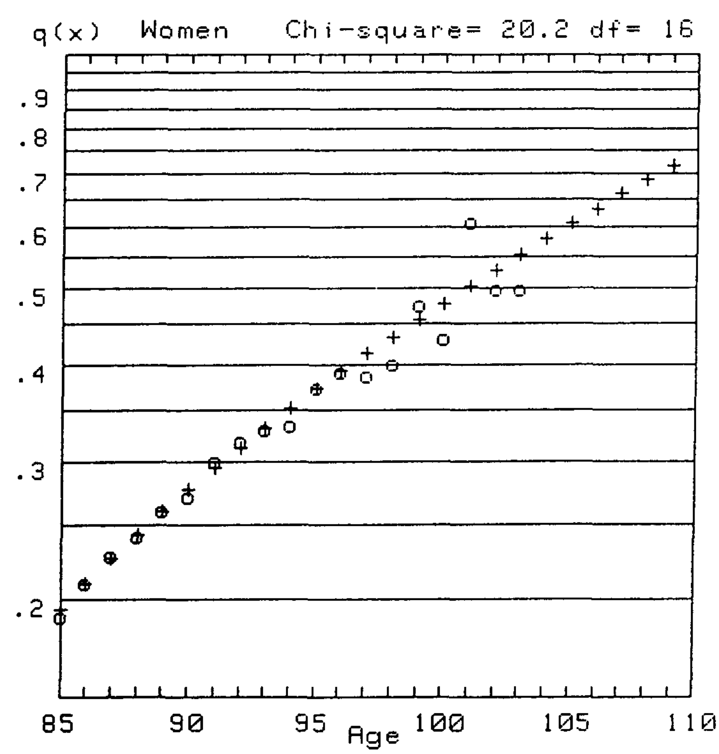
Switzerland 1876-1914



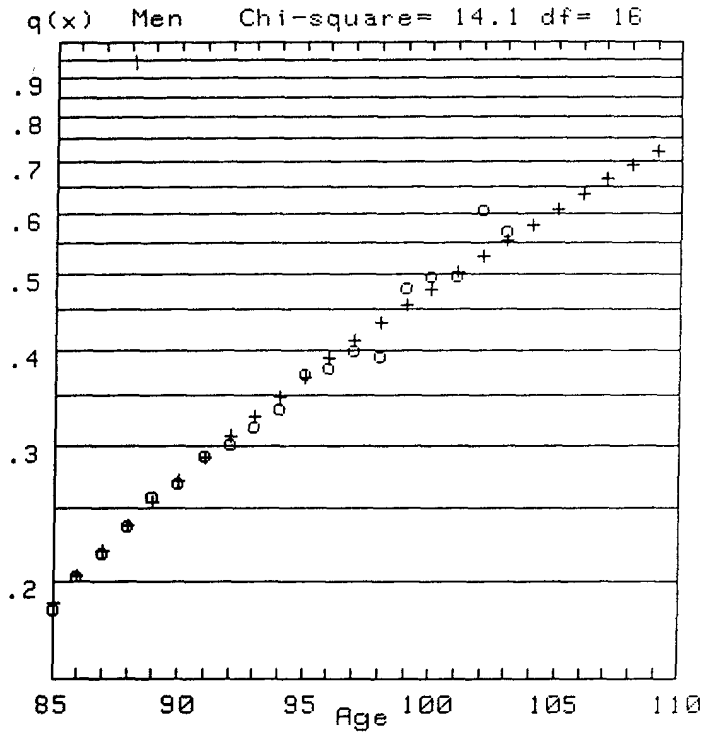



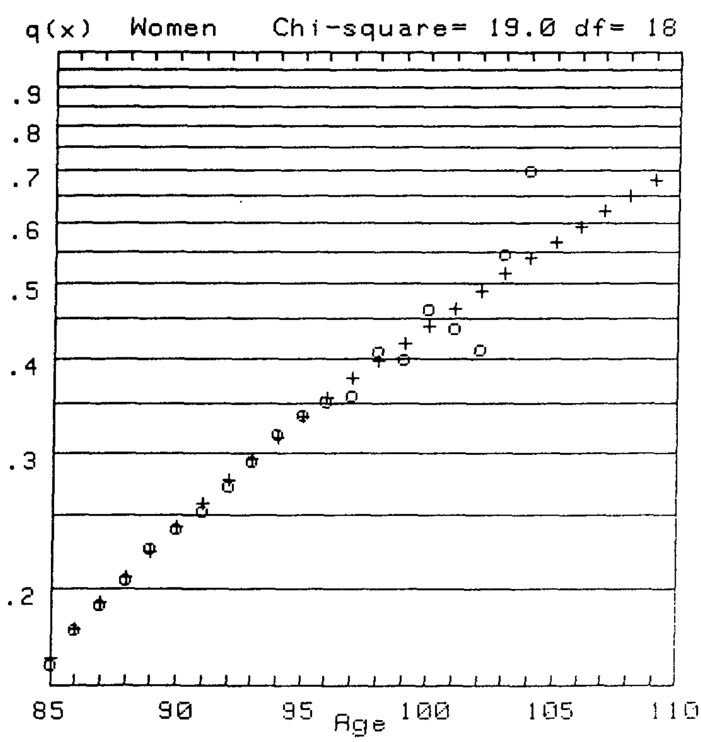
Switzerland 1914-1948



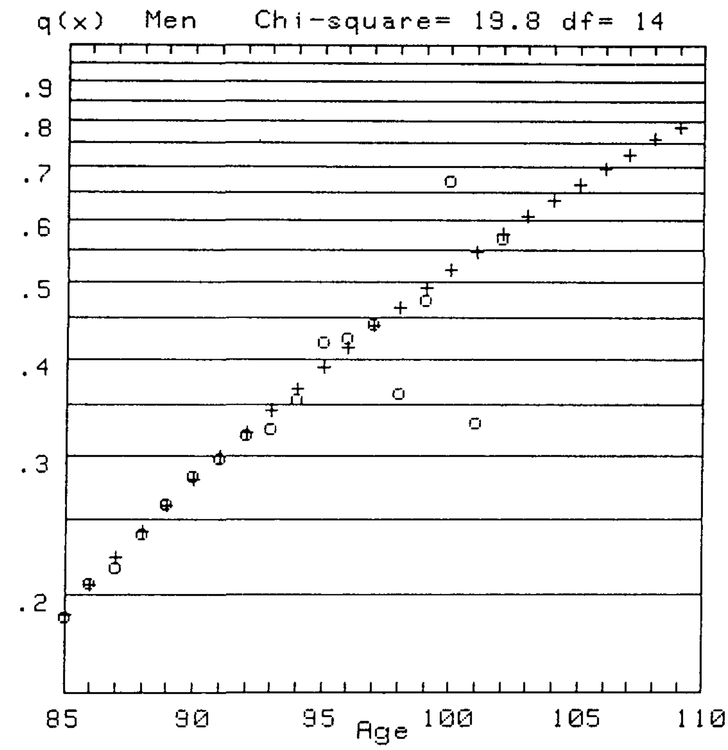



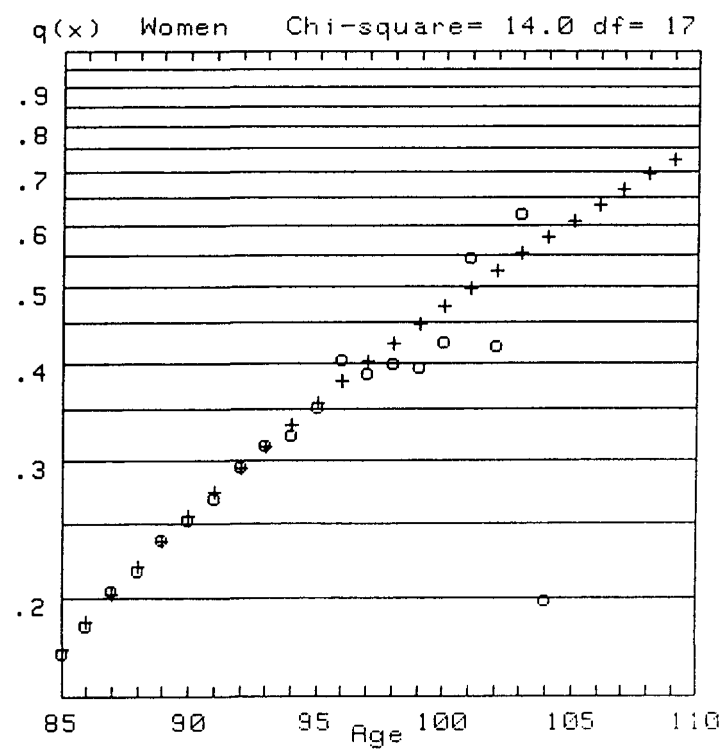
Switzerland 1948-1970



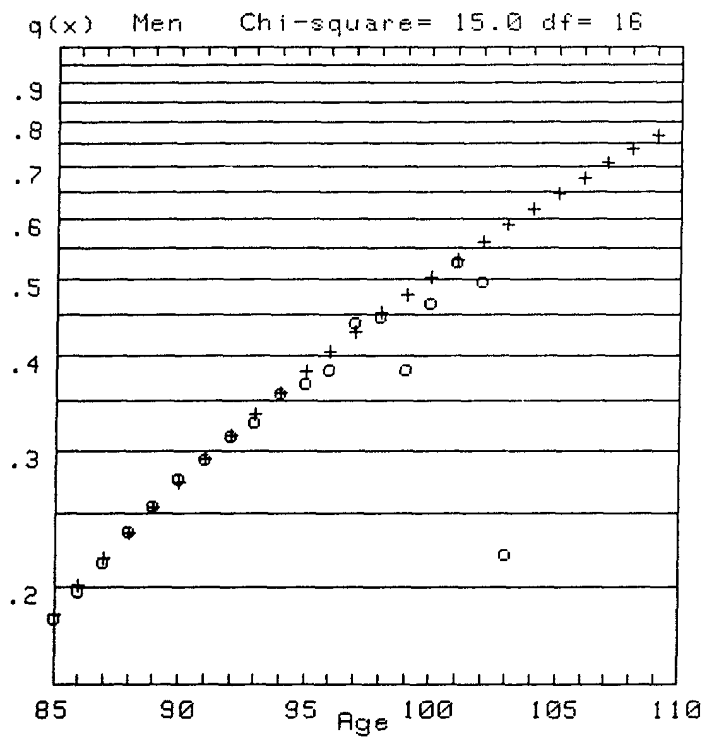



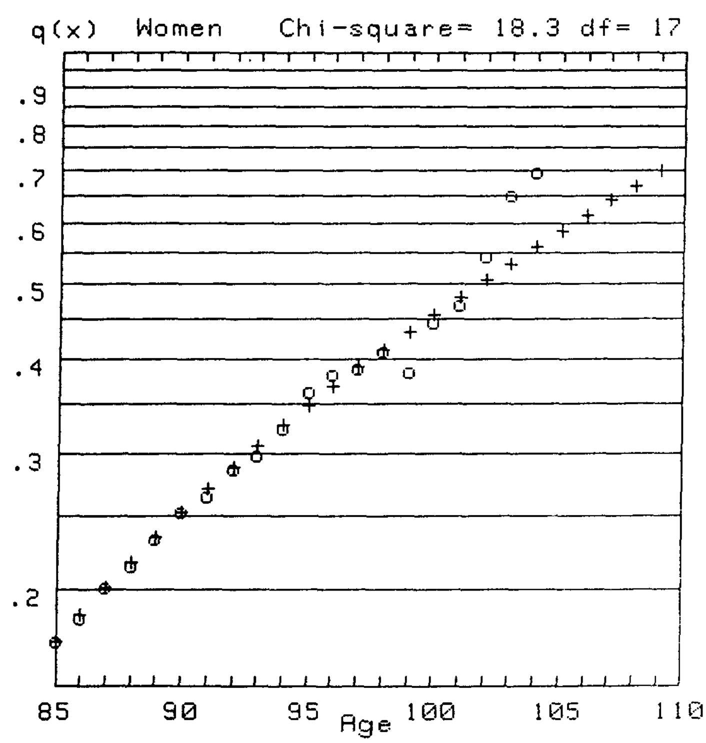
Sweden 1901-1914



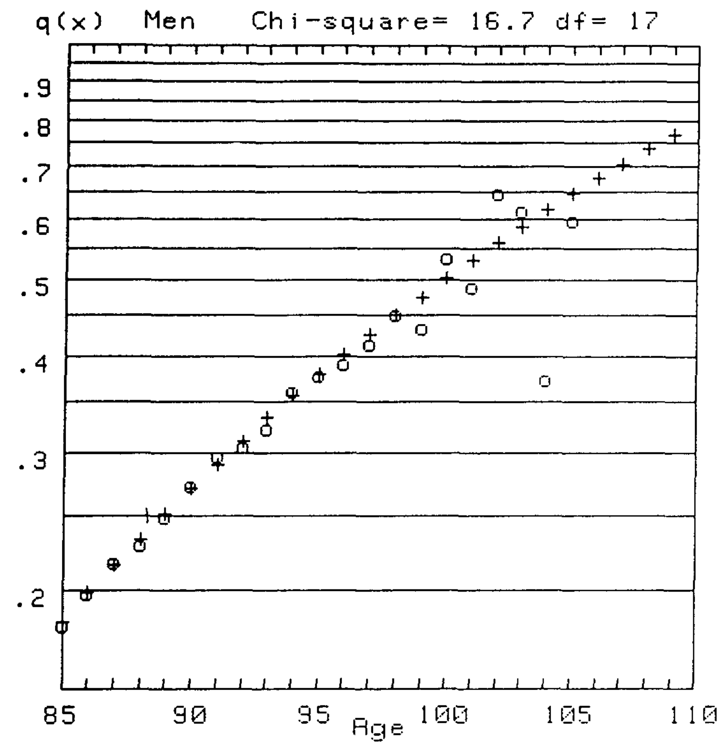



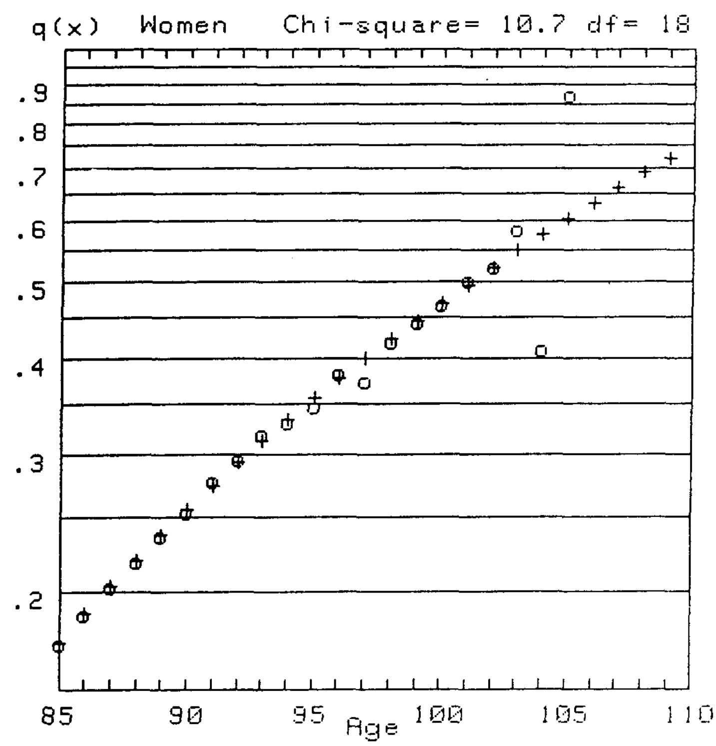
Sweden 1914-1930



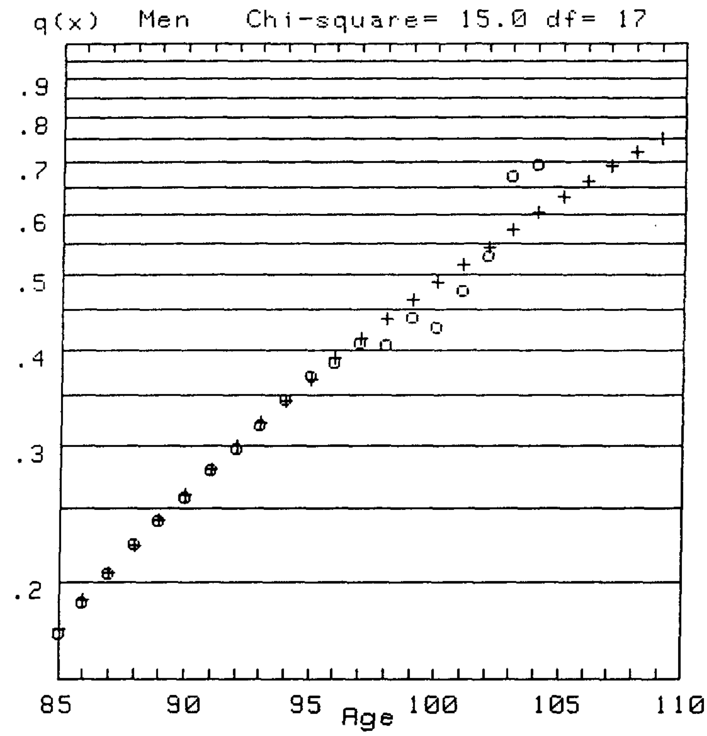



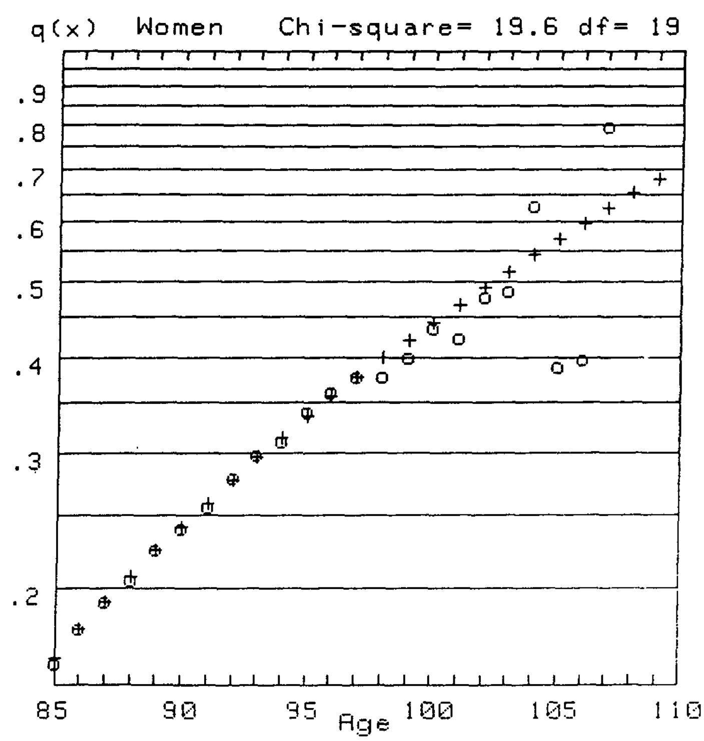
Sweden 1930-1945



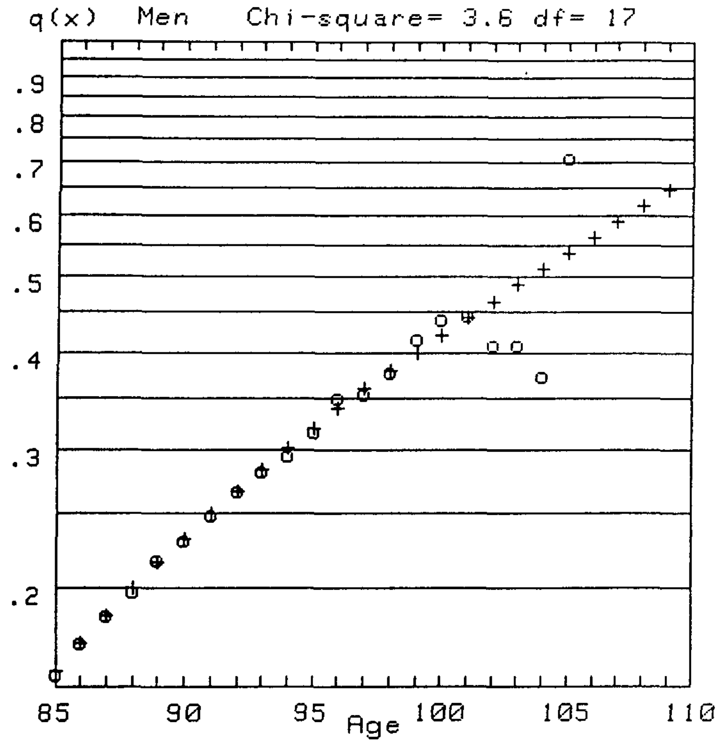



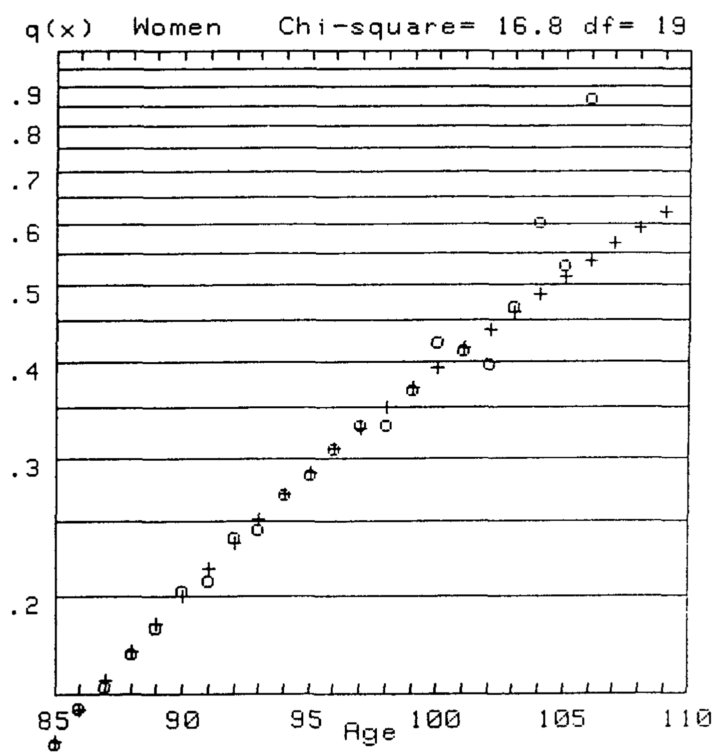
Sweden 1945-1967



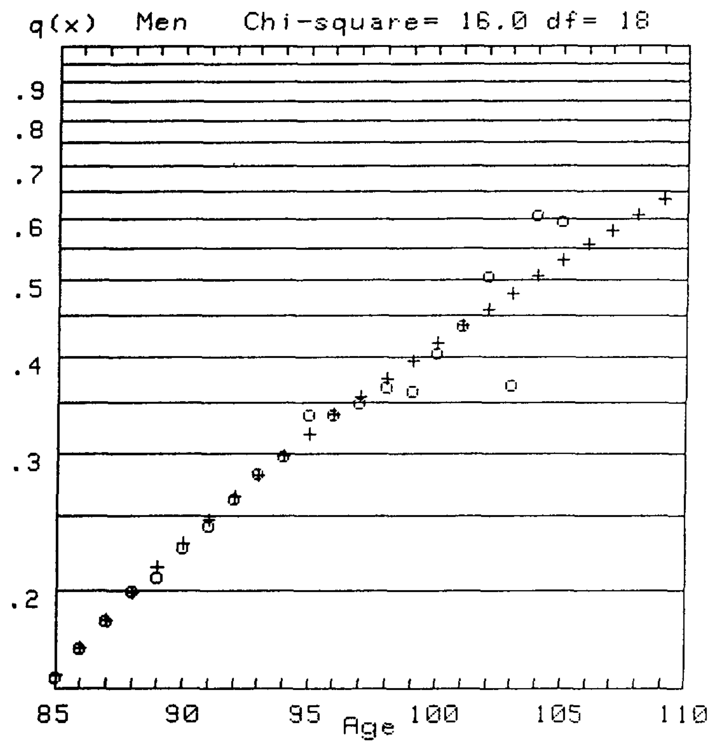



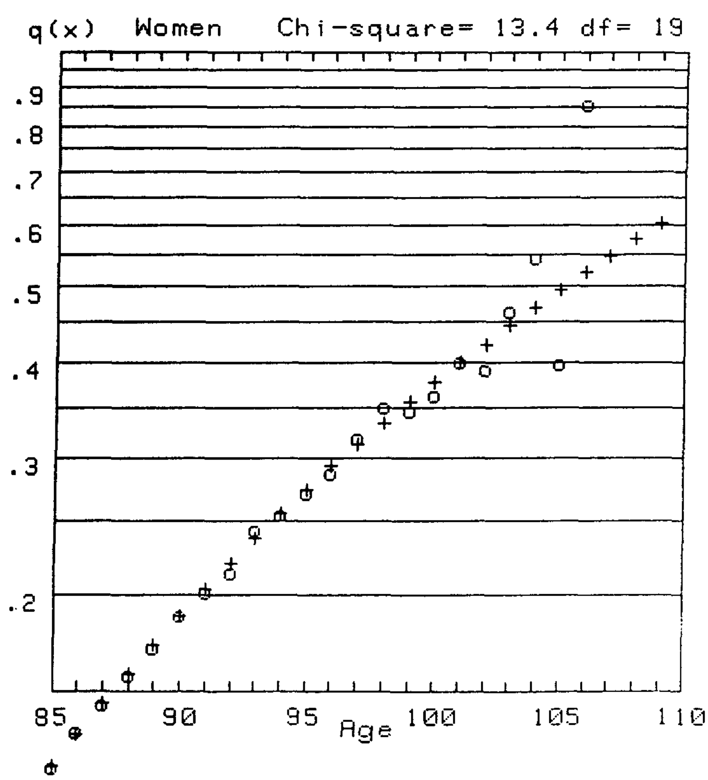
Sweden 1967-1973



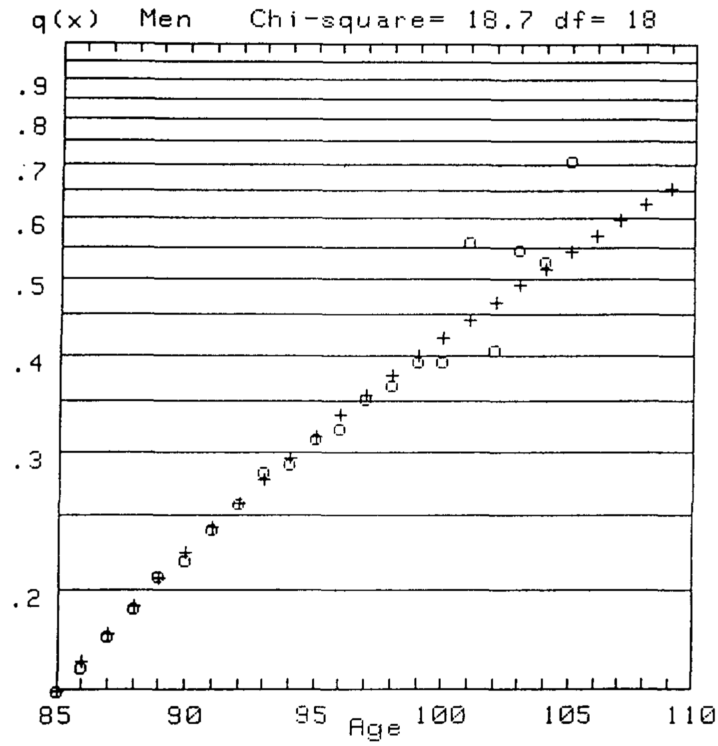



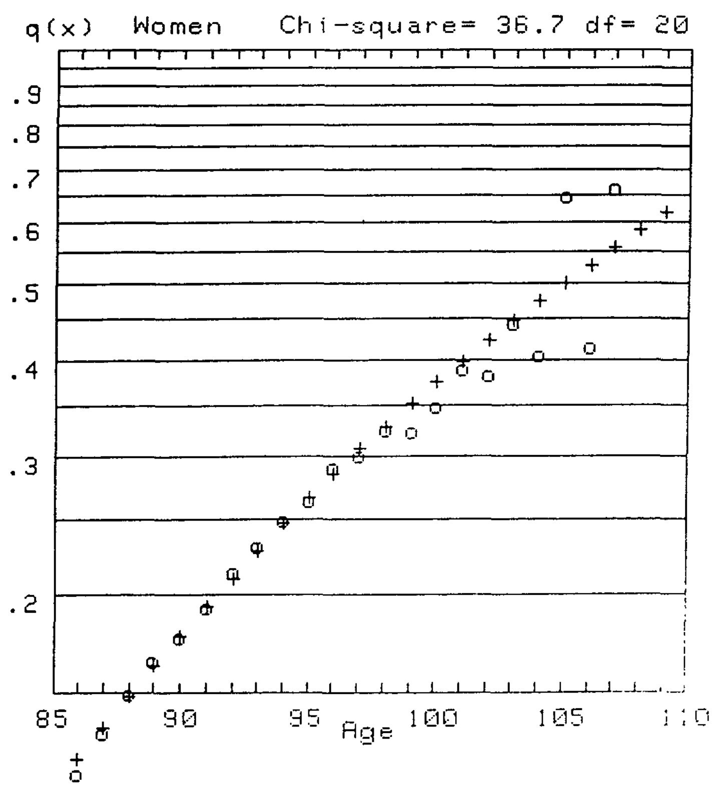
Sweden 1973-1979



Sweden 1979-1984



Tidigare nummer av Promemorior från U/STM:
NR

1 Bayesianska idéer vid planeringen av sample surveys. Lars Lyberg (1978-11-01)

2 Litteraturförteckning över artiklar om kontingenstabeller. Anders Andersson (1978-11-07)

3 En presentation av Box-Jenkins metod för analys och prognos av tidsserier. Åke Holmén (1979-12-20)

4 Handledning i AID-analys. Anders Norberg (1980-10-22)

5 Utredning angående statistisk analysverksamhet vid SCB: Slutrapport. P/STM, Analysprojektet (1980-10-31)

6 Metoder för evalvering av noggrannheten i SCBs statistik. En översikt. Jörgen Dalén (1981-03-02)

7 Effektiva strategier för estimation av förändringar och nivåer vid föränderlig population. Gösta Forsman och Tomas Garås (1982-11-01)

8 How large must the sample size be? Nominal confidence levels versus actual coverage probabilities in simple random sampling. Jörgen Dalén (1983-02-14)

9 Regression analysis and ratio analysis for domains. A randomization theory approach. Eva Elvers, Carl Erik Särndal, Jan Wretman och Göran Örnberg (1983-06-20)

10 Current survey research at Statistics Sweden. Lars Lyberg, Bengt Swensson och Jan Håkan Wretman (1983-09-01)

11 Utjämningsmetoder vid nivåkorrigering av tidsserier med tillämpning på nationalräkenskapsdata. Lars-Otto Sjöberg (1984-01-11)

12 Regressionsanalys för f d statistikstuderande. Harry Lütjohann (1984-02-01)

13 Estimating Gini and Entropy inequality parameters. Fredrik Nygård och Arne Sandström (1985-01-09)

14 Income inequality measures based on sample surveys. Fredrik Nygård och Arne Sandström (1985-05-20)

15 Granskning och evalvering av surveymodeller, tiden före 1960. Gösta Forsman (1985-05-30)

16 Variance estimators of the Gini coefficient - simple random sampling. Arne Sandström, Jan Wretman och Bertil Waldén (Memo, Februari 1985)

17 Variance estimators of the Gini coefficient - probability sampling. Arne Sandström, Jan Wretman och Bertil Waldén (1985-07-05)

18 Reconciling tables and margins using least-squares. Harry Lütjohann (1985-08-01)

19 *Ersättningens och uppgiftslämnarbördans betydelse för kvaliteten i undersökningarna om hushållens utgifter*. Håkan L. Lindström (1985-11-29)

20 A general view of estimation for two phases of selection. Carl-Erik Särndal och Bengt Swensson (1985-12-05)

21 On the use of automated coding at Statistics Sweden. Lars Lyberg (1986-01-16)

22 Quality Control of Coding Operations at Statistics Sweden. Lars Lyberg (1986-03-20)

23 A General View of Nonresponse Bias in Some Sample Surveys of the Swedish Population. Håkan L Lindström (1986-05-16)

24 Nonresponse rates in 1970 - 1985 in surveys of Individuals and Households. Håkan L. Lindström och Pat Dean (1986-06-06)

25 Two Evaluation Studies of Small Area Estimation Methods: The Case of Estimating Population Characteristics in Swedish Municipalities for the Intercensal Period. Sixten Lundström (1986-10-14)

26 A Survey Practitioner's Notion of Nonresponse. Richard Platek (1986-10-20)

27 Factors to be Considered in Developing a Reinterview Program and Interviewer Debriefings at SCB. Dawn D. Nelson (1986-10-24)

28 Utredning kring bortfallet i samband med den s k Metropolitdebatten. Sixten Lundström (1986-11-10)

29 Om HINK-undersökningens design- och allokeringsproblematik. Bengt Rosén (1986-10-06)

30 Om HINK-undersökningens estimationsförfarande. Bengt Rosén (1987-01-28)

31 Program för minskat bortfall i SCBs individ- och hushållsundersökningar - förslag från aktionsgruppen för uppgiftslämnarfrågor. Lars Lundgren (1987-02-20)

32 Practical estimators of a population total which reduce the impact of large observations. Jörgen Dalén (87-03-03)

33 A Multivariate Approach to Social Inequality. M Ribe (1987-05-05)

34 BORTFALLSBAROMETERN NR 2. Bortfall vid I-avdelningen, AKU och F-avdelningen t o m 1986. Håkan L. Lindström, Peter Lundquist och Tomas Garås (1987-08-01)

35 Use of Laboratory Methods and Cognitive Science for the Design and Testing of Questionnaires. Handbook of Methods. Judith T. Lessler (June 1987)

36 Some Optimal Problems When Estimating Household Data on the Basis of a "Primary", Stratified Sample of Individuals. Bengt Rosén (1987-08-10)

37 Early Survey Models and Their Impact on Survey Quality Work. Gösta Forsman (1987-09-17)

Kvarvarande exemplar av ovanstående promemorior kan rekvireras från Elseliv Lindfors, U/STM, SCB, 115 81 Stockholm, eller per telefon 08 7834178