

# **A Cohort Model for Analyzing and Projecting Fertility by Birth Order**

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## INLEDNING

### TILL

**R & D report : research, methods, development / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1988-2004. – Nr. 1988:1-2004:2.**

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#### **Föregångare:**

Metodinformation : preliminär rapport från Statistiska centralbyrån. – Stockholm : Statistiska centralbyrån. – 1984-1986. – Nr 1984:1-1986:8.

U/ADB / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1986-1987. – Nr E24-E26

R & D report : research, methods, development, U/STM / Statistics Sweden. – Stockholm : Statistiska centralbyrån, 1987. – Nr 29-41.

#### **Efterföljare:**

Research and development : methodology reports from Statistics Sweden. – Stockholm : Statistiska centralbyrån. – 2006. – Nr 2006:1-.

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**A COHORT MODEL FOR ANALYZING AND PROJECTING  
FERTILITY BY BIRTH ORDER**

by

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1989

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### Summary

In countries with a high degree of family planning the years of childbearing can, without larger changes of completed fertility, be widely displaced within the fertile period. Thus, the decreasing fertility rates during the 1970s can be understood as a postponement of childbearing. A larger and larger part of each generation has chosen to bear children at a later age and, therefore, women in childbearing age will for some years ahead compose a most heterogeneous group with respect to the start of childbearing.

To model fertility for such a heterogeneous population, a cohort model based on birth order data is proposed. The model allows for the fact that younger women show higher progression than older women and that the average length of the birth interval decreases with age. To handle displacement of the childbearing period the notion of 'relative age' is introduced.

The model can be used for short- and medium-term projection of the fertility of a cohort given assumptions about the final parity distribution. The method is applied to Swedish data for projecting the age-specific rate.

## 1. Introduction

In countries with a high degree of family planning the women have a wide choice of the time interval when they want to have their children. Without larger changes of completed cohort fertility, the years of childbearing can be widely displaced within the fertile period. More or less simultaneous advancements or postponements of the childbearing are possible, resulting in large fluctuations in the period fertility level. Well-known are the concentration of the births after the Second World War and in the 1960s.

The decreasing fertility rates during the 1970s can be understood in the same way: The childbearing was postponed. A larger and larger part of each generation has chosen to bear children at a later age, after vocational training and some years of professional work. In Sweden the total fertility rate (TFR) decreased from 2.49 in 1964 to 1.61 in 1983. From this bottom level TFR has steadily increased to 1.84 in 1987 and 1.95 in 1988 (prel. estimate).

A consequence of this process of postponement is that women in childbearing age for some years ahead will compose a most heterogeneous group with respect to the start of childbearing, including the generations of the 1940s, with an early start, as well as the generations of the 1960s, with, on the contrary, an extremely late start. Due to this heterogeneity, the yearly age-specific rates will, in years to come, change systematically towards relatively high rates for women in the second half of the fertile period.

The future changes in the age-specific fertility rates are important for population projections. To predict the future rates assumptions about the final parity distribution of different generations are relevant. And certainly: 'Later births mean fewer births'. But also the current state of childbearing as well as the distribution of birth intervals is relevant. If the final parity distribution of the cohort is compared to the end point of a journey, then the current parity distribution is the starting point and the birth intervals determine the time-table. In this paper a cohort model for analyzing and projecting fertility by birth order is presented. The model is formulated in the well-known terms of progression ratios (directly related to the final parity distribution) and birth intervals. To do so the concepts of 'relative age' and 'progression function' are introduced.

Analyzing and projecting fertility by birth order is no new approach. Thus, fertility projections for England and Wales, based on parity information, have been made by Werner and Chalk (1986). Both parity and duration are included in models by Feeny(1983) and Ní Bhrolcháin (1987). Feeny (1985) also discusses the model from the prediction point of view. But birth cohorts are used only for predicting first births, while the prediction of second and subsequent births is based on the parity cohort - a group of women of the same parity but not necessarily belonging to the same birth cohort - and duration in parity.

But, for several reasons it is important to keep the birth cohorts apart: (1) Younger women in a cohort will in general show higher progression to second and subsequent births than older women of the same cohort. (2) The average length of the birth interval decreases with age. (3) To be able to state different assumptions about the fertility of the birth cohorts. Therefore, a cohort model for fertility by birth order has been developed, formulated in terms of age-order-specific rates.

## **2. Birth order data in Sweden**

Although registration of vital statistics has a long tradition in Sweden, collecting information about the mother's number of previous births is of a rather late date. It was introduced for marriage cohorts in 1950: Only births within marriage were then registered. These data were soon found inadequate, mainly because the first child often was born before the marriage. In 1973 the registration was changed to include the number of all live births to the mother and the date of the last previous birth. But as the observation period is short, the data is of limited value for analyzing cohort fertility by birth order. All the same, vital statistics on birth order, covering the fertility periods of women born 1925 and later, are available due to the personal identity number.

Using the personal identity number, census data and vital statistics from different years have been merged to a fertility register, containing longitudinal birth data for female generations born 1925 and later. Women, who were recorded in the 1960 population census or born in Sweden thereafter, have been followed until the end of 1987. Information about the



children of these women is partly based on the census (children born 1943 - 1960) and partly on vital statistics ( children born after the census). See Johansson and Finnäs(1983).

Completed cohort fertility computed on vital statistics for all women, including immigrants after 1960, is slightly higher than for women in the fertility register. The population of women in the register is almost closed: There is only a slow decrementation due to emigration and mortality. Since women who emigrated before the 1960 census were never included in the register, observed emigration is less frequent than it should have been if the cohorts had been followed from the year of birth.

The analysis by Johansson and Finnäs was based on the register data up to the end of 1977. Due to the somewhat dubious quality of the data on early generations, only women born 1930 and later are considered. Qvist(1987) follows the same women to the end of 1985. We summarize below the main findings for cohorts with a completed fertility period, i.e., women born 1930 - 1940:

- the final childlessness varies between 12 and 14 percent
- the proportion of women continuing from a first to a second birth shows a slow increase from 78 to 82 percent
- the proportion of women continuing to a third birth has decreased and the same is true for women progressing to a fourth birth.

Concerning later generations, the characteristic feature of childbearing has been the extent to which women have delayed the birth of their first child. The age at which half of the women have given birth to their first child has increased about three years from the generations of the mid 1940s to the generations of the late 1950s. The number of childless women in fertile age has therefore been increasing for several years. Does this change in the fertility pattern just mean a postponement of childbearing or does it imply fewer children per women? If we should take stated birth plans seriously (Nordenstam (1984)), there will be no significant reduction in the final family size of these young generations. To some extent this may be true, because at moderate levels of fertility the women have a fairly wide choice of the period when they would like to bear the children.

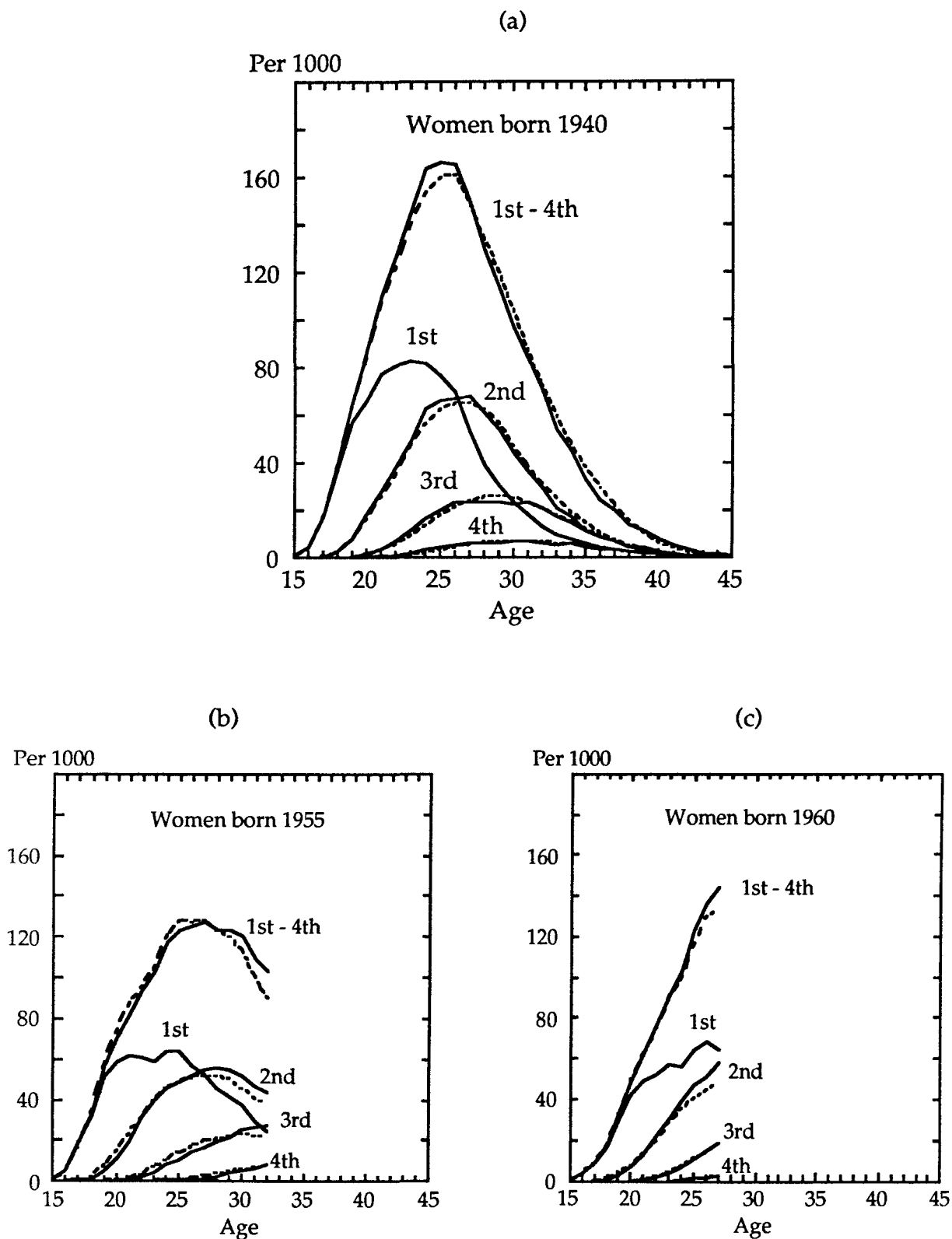


Fig 1a-c. Actual and predicted (dotted) age-order-specific rates of second through fourth births. The first order rates are not predicted. All predicted rates are based on the progression ratios of the 1940 generation. Number of births per 1000 women.

### 3. About age-order-specific birth rates

#### 3.1 Definitions and some useful functions

We define, in accordance with Shryock , Siegel et al. (1973, p 475), the  $p$ :th order birth rate at age  $x$  in a certain cohort as  $f_p(x) = B_p(x)/M(x)$ , where  $B_p(x)$  is the number of births of order  $p$  and  $M(x)$  is the mean population at age  $x$  - age at the end of the calendar year. Fig. 1a shows these rates for women born in 1940 ( $p = 1, \dots, 4$ ). As births of order five are rather unusual in Sweden nowadays and are supposed to be rare in the future, we will consider only the first through the fourth births.

The age-specific fertility rate is the sum  $f(x) = \sum_p f_p(x)$  which is also drawn in the figure. Another useful function of the age-order-specific rates is the parity total:  $f_p = \sum_x f_p(x)$ , which is the estimated proportion women who give birth to a child of order  $p$  during the fertility period. The total fertility rate is:  $TFR = \sum_x f(x) = \sum_p f_p$ . An estimate of final childlessness is given by  $1 - f_1$ .

#### 3.2 The development in the age-specific rates of first births

The first child marks the beginning of the family building process, while second and subsequent children are more or less a consequence of the first one. The development in the age-specific rates of first births explains, therefore, a large part of the time variation in fertility during the last decades.

Fig 2 shows yearly first order birth rates, based on women born 1925 and later. As the data is collected for cohorts, the time-series differ in length. A striking feature of the graphs is the clear dependence on time and the parallel development of the series. A fast decrease in the rates after the boom in the mid 1960s, especially for those aged 18 - 26, is conspicuous. From that time, data indicates a more than a 10 year long period of postponement of births. The peak for women aged 21 - 25 in the first half of the 1970s is mainly a consequence of the fast decrease in teenage fertility a few years before. The displacement of birth to later ages seems to have

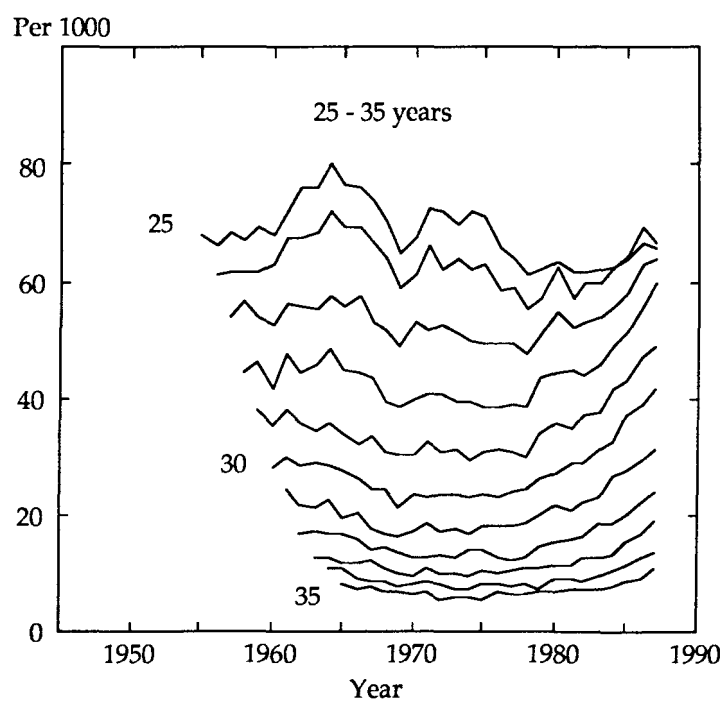
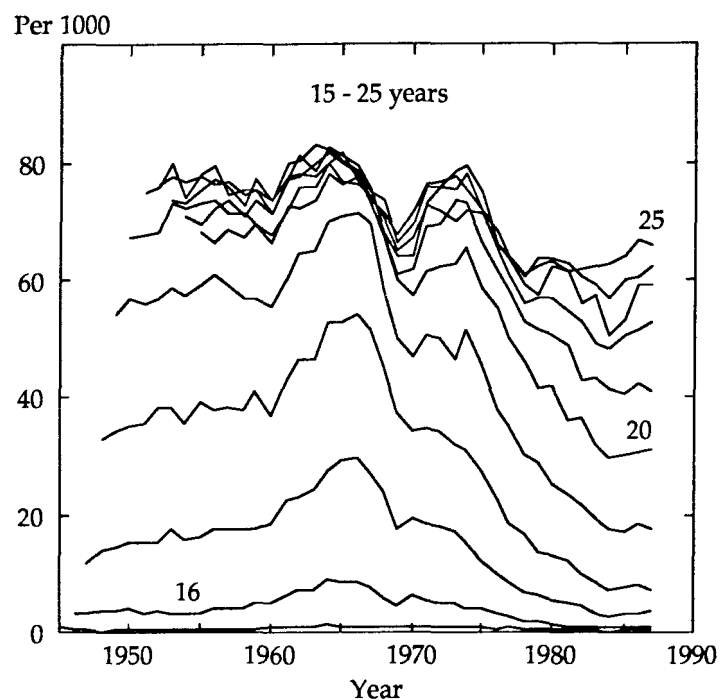


Fig 2a - b. First order age-specific birth rates in Sweden based on the fertility register. Number of births per 1000 women.

terminated around 1984, where the rates have stabilized. The increase in ages 25 - 35 is an effect of the postponement. Below it is shown how the birth rates of second and subsequent order can be related to the rates of first births.

## 4. Modelling age-order-specific rates

### 4.1 Relative age and progression functions

To allow for displacements of the childbearing period within fertile age, the notion of 'relative age at the  $p$ :th birth' is introduced, denoted  $z_p$ , which is defined as the cumulative proportion of completed births of order  $p$  ( $z_p = G_p(x)$ ;  $0 < z_p \leq 1$ ). If, for instance, a woman has the relative age  $z_2$  close to zero, then this means that she is among the first women of her generation to have a second child. By  $z_p$  we get an ordering of the ages at the  $p$ :th birth. The index  $p$  will be omitted when there is no risk of confusion.

The proportion of the women with one child who give birth to a second child depends on relative age  $z_1$ . The proportion is larger for younger women than for older women in the same cohort. We could speak of a varying progression ratio, but the term 'progression ratio' is reserved for the overall proportion. The word 'progression function' will be used, instead. As the same is true for progression from second and subsequent births, there is a progression function for each parity, denoted  $\phi_p(z)$ , where  $z$  is the relative age defined above. These functions, standardized to a progression ratio of one unit, seem to be stable over cohorts and have been estimated for generations with completed fertility (Martinelle 1988). See table 1. The standardization means that we have to multiply by the total progression ratio to get the actual progression function. From the definition of progression function follows that  $\phi_p(z)$  can be seen as a density function and  $\Phi_p(z) = \int_0^z \phi_p(t)dt$  as the corresponding distribution function.

The table shows that  $\phi_1(z)$  is around one unit for almost all  $z$ -values, which means that having a second birth is almost independent of the age at first birth. It is seen that 85 percent of second births are found among women of a relative age of at most 0.80 at first birth ( $\Phi_1(0.80) = .848$ ). Third and fourth births are, on the contrary, more concentrated to lower relative ages: 94 percent of fourth births are found among women of a relative age of at most 0.80 at third birth ( $\Phi_3(0.80) = .941$ ).

**Table 1. Estimated progression functions  $\phi_p(z)$  for  $p = 1, 2$  and  $3$ .**

$z$	$\phi_1(z)$	$\Phi_1(z)$	$\phi_2(z)$	$\Phi_2(z)$	$\phi_3(z)$	$\Phi_3(z)$
.05	1.11	.055	1.69	.087	2.00	.105
.10	1.10	.111	1.59	.169	1.83	.200
.15	1.09	.165	1.49	.246	1.69	.288
.20	1.09	.220	1.40	.319	1.57	.370
.25	1.08	.274	1.32	.387	1.44	.445
.30	1.07	.328	1.25	.451	1.31	.513
.35	1.07	.382	1.18	.512	1.19	.576
.40	1.07	.435	1.12	.569	1.10	.633
.45	1.06	.489	1.07	.624	1.01	.685
.50	1.05	.541	1.01	.676	.94	.734
.55	1.04	.594	.95	.725	.86	.779
.60	1.04	.646	.88	.770	.77	.820
.65	1.03	.698	.83	.813	.69	.856
.70	1.01	.749	.76	.853	.60	.888
.75	.99	.799	.68	.889	.52	.916
.80	.96	.848	.59	.921	.45	.941
.85	.92	.894	.50	.948	.38	.961
.90	.84	.938	.40	.971	.31	.979
.95	.66	.977	.32	.989	.23	.992
1.00	.00	1.000	.00	1.000	.00	1.000

Based on Swedish women born in 1925, 1930, 1935 and 1940.

Relative age ( $z$ ) of order  $p$  is the proportion completed births of order  $p$ .  $\Phi_p(z) = \int_0^z \phi_p(t) dt$

## 4.2 Birth interval distributions

From the prediction point of view, it is also important to know the distribution of birth intervals. The birth interval or the duration is defined as the age difference between successive children, where the age of each child is taken as the age at the end of the calendar year. The length of an interval is, therefore, an integer. It means also, that a birth interval of, say, two years corresponds to a real age difference in the range of one to three years. This may seem imprecise, but is the kind of definition needed for predicting the number of births in steps of one year. Birth intervals of 11 years or more are measured as 10. Because the duration varies with the parity and the age of the mother when the first child in the pair is born, the birth intervals have been grouped by parity and age. Table 2 gives the estimated frequency distributions.

**Table 2. Estimated distributions of birth intervals by age and parity based on Swedish women born in 1925, 1930, 1935 and 1940.**

<b>Parity: 1 -&gt; 2</b>					
Interval (years)	Age				
	-19	20-29	30-34	35-39	40-
0	.4	.8	1.7	3.6	6.3
1	10.0	9.2	7.3	11.3	15.6
2	22.8	22.0	22.8	29.8	38.1
3	18.0	23.3	25.9	25.7	28.8
4	13.6	16.3	18.2	14.7	6.9
5	9.3	9.8	10.8	7.2	3.1
6	7.6	5.9	5.8	3.9	.6
7	5.0	3.9	3.2	1.9	.6
8	3.5	2.6	2.0	1.1	.0
9	2.4	1.8	1.0	.5	.0
10-	7.4	4.5	1.4	.4	.0
Sum:	100.0	100.0	100.0	100.0	100.0
Mean:	4.1	3.8	3.5	3.0	2.3

<b>Parity: 2 -&gt; 3 and 3 -&gt; 4</b>					
Interval (years)	Age				
	-19	20-29	30-34	35-39	40-
0	.7	1.8	3.9	8.5	25.6
1	11.2	9.6	8.3	11.7	16.7
2	31.5	18.9	18.4	25.5	26.5
3	10.9	15.7	17.5	20.1	18.4
4	10.9	13.5	14.8	14.3	7.3
5	9.7	10.4	11.8	7.5	4.3
6	6.0	8.1	8.4	5.9	1.3
7	4.5	6.0	5.8	3.2	.0
8	3.0	4.4	4.1	1.7	.0
9	4.1	3.3	2.7	.8	.0
10-	7.5	8.4	4.1	.8	.0
Sum:	100.0	100.0	100.0	100.0	100.0
Mean:	4.0	4.4	4.1	3.0	1.8

Based on Swedish women born in 1925, 1930, 1935 and 1940.

### 4.3 The model

The age-order-specific birth rate  $f_{p+1}(x)$  will be predicted by the equation:

$$\hat{f}_{p+1}(x) = \pi_p \sum_{d=0}^{10} \phi_p(\zeta_p) \delta_p(d; x-d) f_p(x-d) \quad (1)$$

where  $\pi_p$  is the progression ratio from parity  $p$  to  $p+1$ ,

$\zeta_p = (G_p(x-d-1) + G_p(x-d))/2$  is the average relative age and  $\phi_p(z)$  and  $\delta_p(d;x)$  are given in Tables 1-2.

The content of the equation is quite simple. To estimate the  $p+1$ :th order birth rate we sum the expected contributions from different years. Each contribution is given by means of the progression function and the birth interval distribution:  $\pi_p \phi_p(\zeta_p) \delta_p(d;x-d)$  is the proportion of the women with the  $p$  birth at age  $x-d$  which is expected to progress from birth of order  $p$  to  $p+1$  and to have the  $p+1$ :th birth at age  $x$ .

The equation cannot be used for predicting the age-specific rate of first births ( $f_1(x)$ ). But, it gives us the estimated rate of second births from the observed rate of first births ( $f_1(x) \rightarrow \hat{f}_2(x)$ ). In the same way we get  $\hat{f}_3(x)$  from  $f_2(x)$ , etc. Fig. 1a shows both observed and predicted (dotted line) age-order-specific birth rates for the 1940 generation.

Actual and predicted age-order-specific rates are quite close. Deviations exist, of course, but they rather confirm than reject the model. Thus, the actual rates are above the predicted ones during the baby boom in the mid 1960s and below thereafter. The same time pattern is displayed by the deviations for other cohorts (Martinelle(1988)), indicating that model errors are mainly of temporal nature.

## 5. Using the model for standardized fertility comparisons

The above model can also be used for analyzing the fertility of cohorts, which have not yet completed their fertility period. Take, for instance, the 1955 generation. The age-order-specific rates until the end of 1987 are shown in Fig. 1b. Estimated childlessness at age 32 is 24 per cent. The corresponding figure for the 1940 generation is 17 per cent. What can then be said about the fertility of women born in 1955 if we allow for the delay of first births? Is it just a question of a delayed first child with an automatic delay of the



following children or have the births of second and subsequent orders also changed radically?

To answer the question we predict the age-specific rates of second to fourth births, using the progression ratios of the 1940 generation. The predicted rates are also shown in Fig. 1b. We see that the second-order birth rate is below the predicted rate up to age 25, but after that age most of the delayed births seem to have been compensated. By summing actual and predicted age-order-specific rates over age we get the following standardized numerical comparison of cumulative fertility at age 32 between the two cohorts.

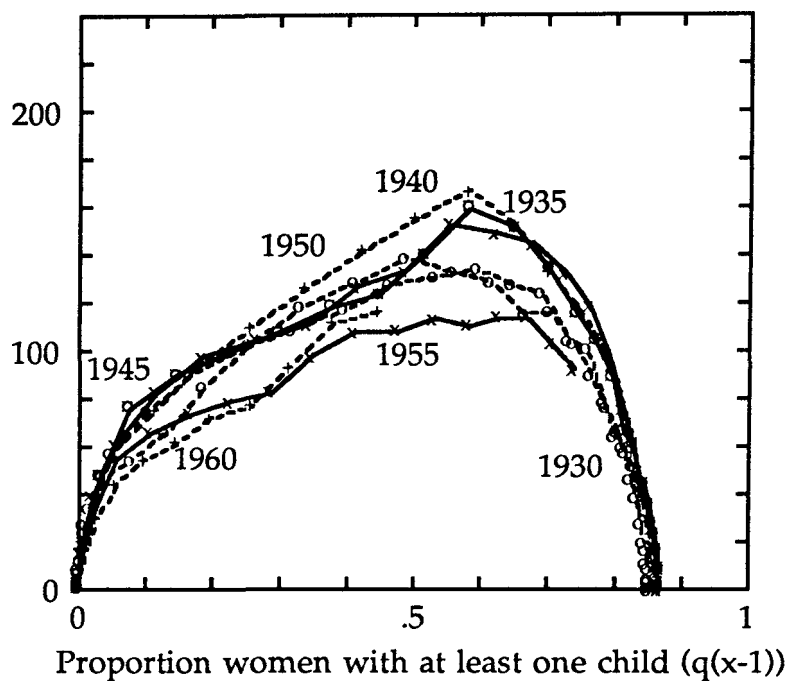
Birth order	Estimated proportion of births for women born in 1955 based on	
	actual rates	predicted rates
1	.76	(.76) <sup>1)</sup>
2	.56	.55
3	.18	.19
4	.03	.04
Sum	1.53	1.54

1) Not predicted, but included in the sum

We see that it is a good agreement between actual and predicted proportions. The average number of children per woman up to age 32 is 1.53, just slightly less than the predicted value of 1.54. In other words: So far, the 1955 generation just seems to have postponed births. But what will happen with the birth rates after age 32?

Of course, we have no definite answer to the question. The best we can do, is to predict the future birth rates, assuming the final fertility level to be known and we can do it for different plausible alternatives. First, we must, however, tackle the problem of predicting the first-order birth rate corresponding to a given percentage of final childlessness.

Number of first births  
per 1000 childless women



Number of first births  
per 1000 childless women

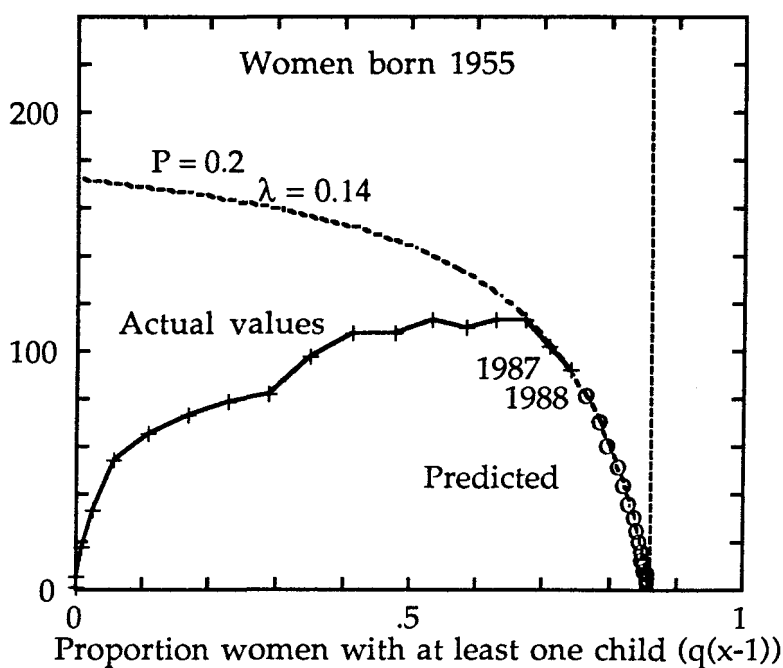


Fig 3a-b. The yearly number of first births per 1000 childless women by the proportion of women with at least one child:(a) For every fifth generation from 1930 to 1960. (b) For women born 1955.

## 6. On the projection problem

### 6.1 How to predict the timing of the first child?

We want to predict the age-specific rates of first births in a cohort for, say,  $x > x_0$ . The problem is that we do not find the kind of constraints which relate second and subsequent births to first births. Earlier, for very short-term prediction the number of newly married could be used, but not anymore, because of the large proportion of extra-marital births. Instead, we have to look for statistical models which just make use of the regularities in the first order birth rates.

Bloom (1982) proposed Coale's marriage model for projection of first order birth rates, but, unfortunately, it does not seem to be of much help. The start of childbearing has in later generations spread out during the fertile period. The cohorts have become more heterogeneous than before with regard to the timing of the first birth, and to such an extent that we think it would be necessary to use a mixture of such distributions. But, that kind of approach would be unduly complicated. Instead, we have chosen to question the role of age.

Why should we relate the first birth rate to age? What really matters, at least in a large part of the fertile period, is the degree of childlessness or, inversely, the proportion of women with at least one child. Therefore, the yearly proportion of childless women who give birth to a first child has been plotted against the proportion non-childless women (Fig 3a). More precisely, denoting the estimated proportion of women with at least one child at age  $x$  by  $q(x)$ , i.e.  $q(x) = f_1(15) + \dots + f_1(x)$ , the function  $f_1(x)/(1-q(x-1))$  has been plotted against  $q(x-1)$  for every fifth generation from 1930 to 1960. All curves start in origin and end - in case of completed fertility - in almost one and the same point, the point corresponding to final childlessness, which, in these cohorts, varies between 12 and 14 percent. The different paths between the two points look almost like ballistic curves, with varying elevation and height, but with a very regular last part approaching the target.

This pattern can be explained by assuming each female generation to contain fertile and non fertile women, including in the last group of women who are fertile but want to stay childless. Let the proportion non fertile women be  $\lambda$  in the cohort and let each fertile and childless woman aged  $x$  have the probability  $P(x)$  to get her first child during a period of one year.

The expected number of first births at age  $x$  is then  $N_x(1 - q(x-1) - \lambda)P(x)$ , where  $N_x$  is the cohort size at age  $x$  - migration and mortality disregarded. As the number of childless women entering age  $x$  is  $N_x(1 - q(x-1))$ , we get the expected proportion women with first births at age  $x$ ,  $g(x)$ , as:

$$g(x) = \frac{N_x(1 - q(x-1) - \lambda)}{N_x(1 - q(x-1))} P(x) = \left(1 - \frac{\lambda}{1 - q(x-1)}\right) P(x) \quad (2)$$

If we put  $P(x) = 0.2$  for all  $x$  in the entire fertility interval and  $\lambda = 0.14$  then the relation between the expected proportion of first births and the proportion of women with at least one child is as the curved and dotted line in Fig. 3b. This simple model with constant birth probability is useful for prediction of the birth rates: We get  $\hat{f}_1(x) = g(x)(1 - q(x-1))$  and, since  $q(x) = q(x-1) + f_1(x)$ , the rates can recursively be computed. This has been done for women born 1955. The predicted  $g(x)$ -values ( $x = 33, 34, \dots, 45$ ) are shown in Fig 3b and the corresponding first order birth rates in Fig 4.

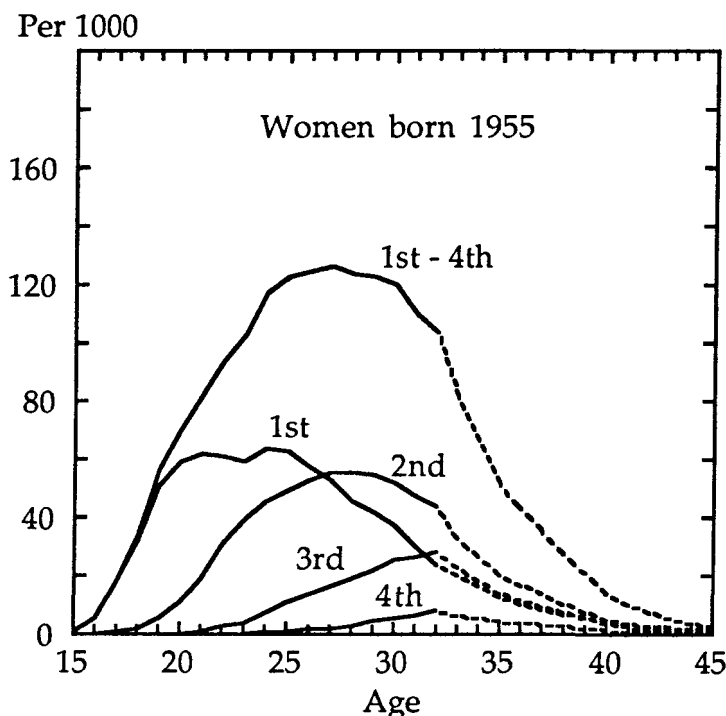


Fig 4. Age-order-specific rates for women born 1955. Predicted rates after age 32:  $\pi_0 = .86$ ,  $\pi_1 = .82$ ,  $\pi_2 = .39$  and  $\pi_3 = .23$ , i.e. TFR = 1.90. Number of births per 1000 women.

A constant probability of first birth (  $P(x) = \text{constant}$  ) among fertile childless women gives an acceptable description of actual data only to the right, near the end point. At lower ages we have found that a good description of the birth rates is obtained by assuming the probability to increase linear with age up to a limit of  $P(x) \approx 0.2$  , corresponding to the dotted line in Fig 3b. This regularity of the  $P(x)$ -values can be used for prediction of the first order birth rates in cohorts born after 1955. The computation is analogous with the case of constant probability. First  $P(x_0)$  must be estimated, which can be done by substituting  $f_1(x_0)/(1 - q(x_0 - 1))$  for  $g(x)$  in equation (2) and solving for  $P(x_0)$ . A yearly increase in the probability of .015 gives a good description of the 'paths' of later cohorts.

## 6.2 Prediction of the total age-specific fertility rate

Assuming the fertility level (in terms of the progression ratios:  $\pi_0, \dots, \pi_3$  ) and the predicted age-specific rate of first births to be given, we apply the prediction equation recursively for  $x > x_0$  :

$$\hat{f}_1(x) \rightarrow \hat{f}_2(x); \hat{f}_2(x) \rightarrow \hat{f}_3(x), \text{ etc.}$$

To get a smooth connection between the actual rates ( $x \leq x_0$  ) and the predicted rates ( $x > x_0$  ), the errors  $e_p(x) = f_p(x) - \hat{f}_p(x)$  , cf. Fig 1b, have been extrapolated. These errors are positively autocorrelated and can be predicted more or less sophisticatedly. We have just put  $e_p(x_0 + 1) = 0.67 e_p(x_0)$  and  $e_p(x_0 + 2) = 0.33 e_p(x_0)$  , which have been added to  $\hat{f}_p(x_0 + 1)$  and  $\hat{f}_p(x_0 + 2)$  respectively to give new estimated rates . In each step we have adjusted these rates by a factor in order to fulfill the constraints  $f_{p+1} = \pi_p f_p$  ( $p = 0, \dots, 3$  and  $f_0 = 1$  ).

The predicted rates for women born in 1955 are shown in Fig. 4. The age-specific fertility rates are obtained as the sum of the age-order-specific rates.

## 7. A possible improvement of the model

The age-order-specific birth rates estimated by the model (1) give a good approximation of actual rates. Deviations occur and are obviously not random. See Fig 1. Thus, for women born 1940 the actual rates exceed the estimated rates around age 25, i.e. during the boom in the 1960s. For the 1955 generation the opposite is true and for women born 1960 the situation is

reversed once again. Concerning the last two generations, the final parity distribution is not known. But only a small part of the deviations is probably due to erroneous progression ratios. A much larger part can be attached to a changing birth interval distribution. During the first half of the 1960s first birth rates were high and the distance between the children were small. In the 1970s, with declining first birth rates, the average birth interval increased, particular for women around 20, resulting in a delay in births of second and subsequent births. During the last few years these higher order rates have surpassed the estimated rates for practically all generations in fertile ages. Delayed births is one reason, but advancements of second and third births may be the case for later generations.

In view of what has been said about the deviations above, in order to get better predictions it would be worth while to improve the description of the birth interval distribution. This can be done by making the description of the age dependence more detailed ( e.g. splitting the interval 20 - 29 years in Table 2) and perhaps also by allowing the distribution to change over time, which, in fact, it has done during the 1960s and the 1970s and perhaps also during the last years.

## Summary

In countries with a high degree of family planning the years of childbearing can, without larger changes of completed fertility, be widely displaced within the fertile period. Thus, the decreasing fertility rates during the 1970s can be understood as a postponement of childbearing. A larger and larger part of each generation has chosen to bear children at a later age and, therefore, women in childbearing age will for some years ahead compose a most heterogeneous group with respect to the start of childbearing.

To model fertility for such a heterogeneous population, a cohort model based on birth order data is proposed. The model allows for the fact that younger women show higher progression than older women and that the average length of the birth interval decreases with age. To handle displacement of the childbearing period the notion of 'relative age' is introduced.

The model can be used for short- and medium-term projection of the fertility of a cohort given assumptions about the final parity distribution. The method is applied to Swedish data for projecting the age-specific rate.

## References

- Bloom, D.E.(1982):Age Patterns of Women at First Birth. *Genus* ,38, pp.101 - 128 .
- Feeney,G.(1983):Population Dynamics Based on Birth Intervals and Parity Progression. *Pop. Studies*,37, pp 75 - 79 .
- Feeney,G.(1985):Parity Progression Projection. *IUSSP 1985*, Florence, Vol 4, pp 125 -136 .
- Johansson, L. and Finnäs, F.(1983):*Fertility of Swedish women born 1927-1960. Urval*, Statistics Sweden.
- Martinelle, S.(1988):‘A cohort model for fertility prediction’. Bakgrundsmaterial 1988:1, Population Research Office, Statistics Sweden.
- Ní Bhrolcháin, M.(1987): Period Parity Progression Ratios and Birth Intervals in England and Wales, 1941 - 1971: A Synthetic Life Table Analysis. *Pop. Studies*, 41, pp 103 - 125 .
- Nordenstam, U.(1984):*Ha barn - men hur många? Information i prognosfrågor* 1984:4, Statistics Sweden.
- Qvist, J. (1987): *Kohortfruktsamhet 1945 - 1985. Demografiska rapporter* 1987:3, Statistics Sweden
- Shryock, H., Siegel, J. and Associates.(1973): *The Methods and Materials of Demography*, U.S. Bureau of the Census.Washington, D.C.
- Werner, B. and Chalk, S.(1986): Projections of first, second, third and later births. *Pop. Trends*, 46, pp 26 - 34 .

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