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VARIATIONS IN THE AGE-PATTERN OF FERTILITY IN SWEDEN AROUND 1986

by

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Abstract: The present report gives an analysis of the regional variations in the age-pattern of fertility in Sweden around 1986. Sweden has 24 counties (län) and 284 municipalities (kommuner). In the present report, data on births for the three years 1985, 1986 and 1987 have been merged and related to the corresponding mid-year female populations in 1986. The principal aim of the paper is to show that although fertility now is hovering slightly above and below a net-reproduction rate of 1, there are striking differences in the age-pattern of fertility. It is also shown that although the fertility schedules for the 24 counties vary considerably with respect to shape, the gamma probability density function gives a parsimonious model of current Swedish fertility. In addition, the paper discusses issues relating to when it is justified to graduate the observed age-specific fertility rates. In order to demonstrate the differences between, on the one hand, variations in regional age-patterns at a given point in time and, on the other, temporal changes in the age-pattern, reference is also made to two time series of national fertility schedules. These schedules are for Sweden between 1950 and 1986, and for Australia between 1929 and 1970.

1. INTRODUCTION

The present paper deals principally with the regional variation in the age-pattern of fertility in Sweden around 1986. It is shown that whereas there is relatively little variation between counties in terms of the level of fertility, there is noticeable variation in the age-pattern of fertility. At the municipal level, both the level and age-pattern of fertility vary considerably. The analysis is based on the recorded births during the three-year period 1985-87 and the corresponding mid-year populations of women. Results are given for the 24 counties (län) and the 284 municipalities (kommuner).

The materials in the present report are meant as a support for studies of the age-pattern of fertility and, especially, as a background material for illustrating the usefulness of the gamma probability density function as a model of fertility.

The discussion of the variation in the age-pattern of fertility is supplemented with two time series of period fertility schedules, namely the 37 annual schedules (given by five-year age groups) for Sweden between 1950 and 1986, and the 42 annual schedules (given by single-year ages) for Australia between 1929 and 1970. The inclusion of these two time series of schedules serves the purpose of demonstrating the essential difference between, on the one hand, regional variation, and, on the other, temporal variation in the age-pattern of fertility.

Issues relating to the graduation of age-specific fertility rates are discussed, and it is shown that the gamma probability density function provides a satisfactory model of current Swedish regional fertility. It is also shown that in the application of the gamma probability density function to fertility, there is noticeable regularity in the estimated parameters.

Section 2 gives the basic definitions and notation used in the paper. Section 3 deals with the variation in fertility patterns. This discussion relates to the counties as well as to the municipalities. Section 4 focuses on when it is justified to graduate the observed age-specific fertility rates. Section 5 shows that the gamma probability density function provides and accurate model of current Swedish period fertility. Section 6 concludes the paper with a discussion.

2. THE AGE-PATTERN OF FERTILITY

2.1 The age-specific fertility rate

To begin with, we introduce the traditional measures of the level and age-pattern of fertility. If B_x is the yearly number of births which takes place among women aged x (in complete years) and E_x is the corresponding mid-year population of women, then

$$f_{X} = \frac{B_{X}}{E_{X}}$$
(1)

is known as the observed age-specific fertility rate for women aged x (it may also be referred to as the observed birth intensity for women aged x). Strictly speaking, E_x is an approximation to the number of person-years lived by women while at age x. In what follows f_x always refers to an observed single-year age-specific fertility rate. To simplify the notation, we write f_x for $f_{x+0.5}$. The set of observed age-specific fertility rates corresponding to the ages 14, ..., 49 is referred to as the fertility schedule and is here denoted $\{f_x\}$.

2.2 The raw moments of the fertility schedule

To summarize the characteristics of a fertility experience, one often makes use of the raw moments of the fertility schedule. Letting

$$R_{i} = \sum_{x} x^{i} f_{x}, \qquad (2)$$

$$R_0 = \sum_{x} f_x$$
(3)

is the total fertility rate, and

$$\mu = R_1 / R_0 \tag{4}$$

is the mean and,

$$\sigma^2 = R_2 / R_0 - (R_1 / R_0)^2$$
(5)

the variance of the fertility schedule. (see Coale and Trussell, 1974, p. 204 and Keyfitz, 1968, p. 141). For simplicity we write R for R_0 . When making numerical calculations, age x in complete years is replaced by exact age x+0.5.

The total fertility rate R is the number of children a woman would bear if she were not subject to the risk of mortality before and during her fecund ages and if her fertility were $\{f_x\}$. Her mean age at childbearing would then be μ . Although, in practice, the mean age of the fertility schedule μ is not the same as the mean age at childbearing (which depends on the age-distribution of women), μ , nevertheless, is a useful measure of the central age of childbearing. The variance σ^2 is a measure of the concentration of childbearing. If the variance is large, then this signifies that women spread their reproduction over a large number of years. If σ^2 is small, then childbearing, by and large, is concentrated to a small range of ages. An additional measure which helps characterize the age-pattern of fertility is the skewness of the fertility schedule given by

$$\gamma = (1/\sigma^3) \sum_{x} (x - \mu)^3 f_x / R$$
(6)

The skewness is a measure of the extent to which the age-pattern of fertility deviates in symmetry about its mean. Fertility curves often, if not always, display positive skewness.

It should be noted that the age-pattern of fertility is best understood in the sense of <u>normalized rates</u>, that is, as the plot of $\{f_X/R\}$ against x. The normalized schedule $\{f_X/R\}$ has a total fertility rate of 1. Implicit in the measures (2) - (6) is a uniform age-distribution of women.

2.3 The ratios of mean parities

The present study makes use of two additional measures of the age-pattern of fertility, namely the ratios of mean parities for the ages 15-19, 20-24 and 25-29 years. Letting

$$F_x = \int_0^x f_t dt$$
,
denote the cumulated fertility at age x, the mean parity for women between 15 and 20 years
is

$$P_1 = (1/5) \int_{15}^{20} F_x \, dx, \tag{7}$$

that is, P_1 is the average number of children born by five women aged 15, 16, 17, 18, and 19, respectively. Similarly, the mean parities for the ages 20-24 and 25-29 are

$$P_2 = (1/5) \int_{20}^{25} F_x \, dx \tag{8}$$

and

20

$$P_3 = (1/5) \int_{25}^{50} F_x \, dx, \tag{9}$$

respectively. It will be noted that the measures (7), (8) and (9) also relate to a uniform agedistribution of women. The ratios of mean parities

$$par1 = P_1/P_2 \tag{10}$$

and
$$par^2 = P_2/P_3 \tag{11}$$

are used to characterize the shape of the normalized fertility schedule.

From a practical point of view, the ratios of mean parities par1 and par2 are estimated by means of (10) and (11) by letting

$$P_{1} = (1/5) (4.5 f_{15} + 3.5 f_{16} + 2.5 f_{17} + 1.5 f_{18} + 0.5 f_{19}),$$

$$P_{2} = S_{1} + (1/5) (4.5 f_{20} + 3.5 f_{21} + 2.5 f_{22} + 1.5 f_{23} + 0.5 f_{24})$$
and
$$P_{3} = S_{2} + (1/5) (4.5 f_{25} + 3.5 f_{26} + 2.5 f_{27} + 1.5 f_{28} + 0.5 f_{29}),$$
(12)

respectively, where
$$S_1 = \sum_{15}^{19} f_x$$
 and $S_2 = \sum_{15}^{24} f_x$.

Whereas the moment measures (2), (3) and (4) are standard ones (see e.g. Keyfitz, 1968, p. 141), par1 and par2 have their roots in indirect estimation of child mortality (see e.g. Brass et al. 1968, pp. 105-122 and Brass, 1975, pp. 50-59) and are intended to measure the rapidity with which the propensity to bear children increases with age (during early fecund ages) and also as indirect measures of the central location of the fertility schedule. Because of their importance in indirect estimation of fertility and child mortality in developing countries, they were included by Coale and Trussell (1974) in their model fertility schedules.

3. REGIONAL FERTILITY IN SWEDEN DURING 1985-87

3.1 Variations in the level of fertility

As noted, the Swedish regional data used in this study are the combined births during the years 1985, 1986 and 1987 and the corresponding mid-year female populations. Table A1 gives the total fertility rate, the mean, variance, skewness, and ratios of mean parities (par1 and par2) for Sweden, the 24 counties and the 284 municipalities. This table as well as other main tables are located in the appendix of the present report. Table A2 which is restricted to the counties gives the same measures as Table A1. In Table A2, however, the counties are sorted in descending order with respect to the total fertility rate and the mean age of the fertility schedule. This table is presented mainly in order to show where each county, relative to other counties, is situated in terms of the level of fertility.

The total fertility rate in Sweden around 1986 was 1.79. The mean age and the variance of the corresponding fertility schedule was 28.92 and 24.94, respectively (Table A1). Letting L_x denote the person-years lived between ages x and x+1 in the Swedish female life table for 1987, and f_x the national age-specific fertility rates for 1985-87 (Table A3), the corresponding net-reproduction rate is

NRR =
$$\sum_{14}^{49} f_X L_X = 1.77 \approx 1.8$$

Thus, given a sex-ratio at birth of about 106 boys per 100 girls, it requires a total fertility rate of about 2.1 to yield a net-reproduction rate of 1 girl child. The Swedish life table for 1987 gives a life expectancy at birth for females of about 80.2 years. Using the Swedish life table for the year 1975, with a female life expectancy of about 77.9 years, gives a net-reproduction rate of $1.76 \approx 1.8$. Hence, recent improvements in the female life expectancy have had a relatively minor effect on the estimated net-reproduction rate.

It would appear, then, that around 1986, the national total fertility rate was about 15 percent lower than that required for replacement. In the most recent national population projections carried out by **Statistics Sweden**, it was assumed that a low fertility alternative would be a total fertility rate of 1.70. The medium alternative was a total fertility rate of 1.83 and the high alternative was one of 1.90. These projections are based on parity progression ratios and lead to proportions of childless women of 20, 15 and 13 percent, respectively. It may well be, however, that Swedish period fertility in the near future recovers to the level of replacement. The total fertility rate for 1987 was 1.85, and births for the first half of 1988 point to a total fertility rate of about 1.95.

Fig. 1 shows the total fertility rate in descending order for the 24 counties. Fig. 1 is based on Table 1 which gives the total fertility rate in descending order as well as the basic fertility measures for the 24 counties. (Thus, the bar above Göteborgs and Bohus län (Fig. 1) refers to Stockholm's län)



Fig. 1.-The total fertility rate for the 24 counties.

It will be noted (Fig. 1) that for all of the 24 counties, the total fertility rate is below the level of replacement. In other words, in each of the 24 counties current fertility, if it were to remain unchanged, would lead to a population decline and an associated increase in the proportion of elderly people¹.

If one were to give the total fertility rates with a precision of one decimal place (Table 1), they would vary between as little as 1.7 and 2.0. In other words there is not much variation in the level of fertility over the counties. With respect to the remaining measures, it is the skewness that varies the most. The remaining measures vary relatively little. The mean varies between 28.3 and 29.6, the variance between 22.2 and 27.5, the skewness between 0.30 and 0.92, par1 between 0.03 and 0.05, and par 2 between 0.20 and 0.28. In essence, it would appear that the variation between the county schedules primarily is due to variations in their variance and skewness.

In several municipalities the total fertility rate is well above what is required for replacement. (As noted, a total fertility rate of about 2.1 is adequate to ensure a net reproduction rate of 1 girl child.) Table 2 gives the 62 municipalities, and their female populations as of January 1, 1988, for which the total fertility rate is at least 2.1.

¹

The argument, of course, assumes that both mortality and fertility remain constant and that the populations are closed to migration.

	Total					
County	fertility rate	Mean	Variance	Skewness	par1	par2
Jönköpings län	1.98	28.58	22.68	0.8279	0.0414	0.2309
Skaraborgs län	1.95	28.45	23.27	0.8772	0.0391	0.2443
Kopparbergs län	1.95	28.69	24.40	0.8265	0.0405	0.2435
Västerbottens län	1.92	29.19	23.65	0.8662	0.0304	0.2033
Kronobergs län	1.91	28.83	22.73	0.7270	0.0457	0.2227
Jämtlands län	1.91	29.03	25.40	0.5417	0.0440	0.2599
Kalmar län	1.90	28.47	23.17	0.9216	0.0345	0.2361
Ävlsborgs län	1.90	28.68	23.41	0.7440	0.0398	0.2393
Blekinge län	1.89	28.26	23.04	0.8502	0.0316	0.2546
Kristianstads län	1.89	28.54	23.41	0.8552	0.0393	0.2389
Hallands län	1.87	29.02	22.19	0.7702	0.0347	0.2071
Östergötlands län	1.86	28.65	24.29	0.7268	0.0447	0.2521
Södermanlands län	1.84	28.25	23.41	0.7530	0.0402	0.2647
Gotlands län	1.84	28.72	24.02	0.6517	0.0414	0.2398
Uppsala län	1.83	29.22	25.65	0.5077	0.0456	0.2538
Norrbottens län	1.83	28.74	24.81	0.8512	0.0368	0.2440
Västnorrlands län	1.82	28.57	22.90	0.7539	0.0408	0.2352
Värmlands län	1.81	28.53	23.95	0.7283	0.0410	0.2552
Örebro län	1.81	28.43	24.38	0.7522	0.0403	0.2708
Västmanlands län	1.81	28.39	23.62	0.8428	0.0330	0.2548
Gävleborgs län	1.78	28.48	24.13	0.7968	0.0347	0.2542
Malmöhus län	1.73	29.02	24.87	0.5350	0.0455	0.2526
Göteborgs och Bohus län	1.71	29.31	25.69	0.3730	0.0457	0.2605
Stockholms län	1.66	29.55	27.45	0.2959	0.0527	0.2779

Table 1.-The total fertility rate, mean, variance, skewness, par1 and par2 for each of the 24 counties around 1986

	Total	Female populatior	Municipality	Total	Female
Ödeshög	2.61	2,940	Kinda	2.21	4,963
Arjeplog	2.61	1,808	Strömsund	2.21	7,763
Bjurholm	2.52	1,432	Lerum	2.20	15,782
Norsjö	2.49	2,647	Vårgårda	2.20	4,762
Ydre	2.42	2,063	Herrljunga	2.20	4,607
Sorsele	2.41	1,722	Vännäs	2.20	4,011
Robertsfors	2.40	3,775	Övertorneå	2.20	2,997
Krokom	2.39	6,707	Älvsbyn	2.20	4,679
Berg	2.39	4,087	Gnosjö	2.19	4,507
Vindeln	2.38	3,226	Sjöbo	2.19	7,473
Aneby	2.35	3,473	Uppvidinge	2.18	5,120
Habo	2.34	4,335	Karlsborg	2.18	3,954
Heby	2.34	6,407	Skinnskatteberg	2.18	2,554
Nordmaling	2.34	3,952	Ragunda	2.18	3,437
Tjörn	2.33	6,230	Håbo	2.15	7,014
Mullsjö	2.33	3,564	Högsby	2.15	3,556
Vilhelmina	2.33	4,090	Öckerö	2.15	5,184
Hylte	2.30	5,252	Orust	2.15	6,338
Säter	2.30	5,659	Borgholm	2.14	5,534
Valdemarsvik	2.28	4,302	Munkedal	2.14	5,277
Sävsjö	2.28	5,713	Laholm	2.13	10,684
Pajala	2.28	3,986	Ekerö	2.11	8,682
Tingsryd	2.26	6,997	Färgelanda	2.11	3,532
Gullspång	2.26	3,092	Munkfors	2.11	2,377
Mörbylånga	2.25	6,411	Östhammar	2.10	10,423
Oskarshamn	2.25	13,728	Gislaved	2.10	14,029
Älvdalen	2.25	3,989	Osby	2.10	6,677
Nordanstig	2.25	5,606	Vara	2.10	8,238
Vaggeryd	2.24	5,964	Hedemora	2.10	8,322
Torsås	2.24	3,750	Bräcke	2.10	4,232
Grästorp	2.23	2,926	Åre	2.10	4,685

Table 2.-The 62 municipalities in which the total fertility rate is adequate for replacement and their female population sizes on January 1, 1988.

The female populations in the above 62 municipalities sum to 331,226 women which means that in terms of the total female population as of January 1, 1988, only about 8 percent of the women lived in municipalities with schedules providing at least full replacement. Generally speaking, the 62 municipalities are in the country side, have small populations, and are in enclaves with sparse large-scale industrial employment.

The 15 municipalities with the lowest total fertility rates are given in Table 3. Among these are the populous city-municipalities of Stockholm, Malmö and Göteborg. On the whole, the municipalities with the lowest fertility have relatively large populations and are situated in the most industrialized and labor intensive parts of the country.

1.69	28,248
1.69	38,639
1.69	8,175
1.69	44,583
1.68	55,936
1.68	33,150
1.67	4,575
1.67	6,849
1.67	29,253
1.63	20,127
1.62	222,763
1.60	121,227
1.57	15,835
1.50	26,606
1.48	356,139
	1.69 1.69 1.69 1.68 1.68 1.67 1.67 1.67 1.67 1.63 1.62 1.60 1.57 1.50

Table 3.-The 15 municipalities with a total fertility rate below 1.70

3.2 Variation in fertility measures

Although for the 284 municipalities, the total fertility rate varies between a minimum of 1.48 and a maximum of 2.61, it is below 2 in 190 (or 67 percent) of the municipalities (Table A2). Consequently, the large majority of the municipality schedules have total fertility rates between roughly 1.5 and 2.0. Within this range, there is considerable variation in the mean, variance and, especially, the skewness of the schedules (Table 4).

Some of the variation is attributable to small population sizes. Even though the births relate to a three-year period, their numbers, nonetheless, are often so small that there is substantial variation in the estimated age-specific fertility rates.

Table 4Measures for 284 Swedish municipality fertility schedules, 1985-87	
	:

Measure	Minimum	Mean	Maximum
R	1.48	1.96	2.61
Mean	27.47	28.54	31.63
Variance	19.08	23.70	33.49
Skewness	0.0306	0.9002	2.3398
par1	0.0038	0.0378	0.1010
par2	0.1018	0.2434	0.3619

3.3 Correlations between fertility measures for the regional schedules

The extent to which the measures are interrelated can be seen by studying their correlations. Table 5 gives the correlation matrix for the six measures and the 284 municipalities shown in Table A2.

The highest correlation is between the total fertility rate and the skewness (r = 0.5793) which indicates that there is a clear tendency for the schedules with high total fertility rates to have a high degree of skewness. However, there is not, as one might have expected, a numerically high correlation between the total fertility rate and the mean (r = -0.1853). The total fertility rate is also virtually unrelated to the measures par1 and par2 indicating that, in so far as the level of fertility is decisive for the shape of the fertility schedule, par1 and par2 do <u>not</u> capture this relationship. Essentially par1 appears to be related to the variance and skewness, and par2 is tied principally to the variance and the mean. The findings concerning par1 and par2 are rather surprising because both par1 and par2 are believed to be "good" indirect measures of the central location of the fertility schedule (see e.g. Brass et al. 1968, pp. 105-122; Brass, 1975, pp. 50-59 and Coale and Trussell, 1974).

	fertility sc	nequies				
=====	========					
	R	Mean	Variance	Skewness	par1	par2
R	1.0000	-0.1853	0.0103	0.5793	-0.0809	-0.0911
Mean		1.0000	0.1685	-0.4372	0.0396	-0.4851
Varian	ice		1.0000	0.0564	0.3381	0.5420
Skewn	ess			1.0000	-0.3416	-0.1477
par1					1.0000	0.3493
par2						1.0000

Table 5.-Correlations between fertility measures for the 284 municipality fertility schedules

Fig. 2 and Fig. 3 show the skewness plotted against the total fertility rate and the mean, respectively. Fig. 4 and Fig. 5 show par2 plotted against the variance and the mean, respectively. Although these figures show clear tendencies of an association between the given measures it is clear, on the other hand, that they do not support the construction of regression representations.

The reason for digressing from the main issue (that of discussing the regional age-patterns of fertility in Sweden around 1986) and alluding to regression representations is, of course, that if the correlations had been higher, it might have been possible to view the selection of regional fertility schedules as a selection of empirical model fertility schedules. One might then have established e.g. a regression representation for estimation of the mean age from a known value of par1 and/or par2 (see e.g. Trussell, 1975). Such a regression representation could then, presumably, have been used in Brass-type estimation of child mortality (Brass, 1975). Nevertheless, even though the schedules are specific for the regional fertility experiences in Sweden around 1986, they yield a fairly large spectrum of age-patterns of fertility and can, no doubt, be used as an illuminating supplement to the Coale-Trussell model fertility schedules.

3.4 Temporal changes in the Swedish level and age-pattern of fertility

As a further illustration of the lack of association between the level and central location in recent Swedish fertility, we show the total fertility rate plotted against the mean for the Swedish annual national fertility schedules between 1950 and 1986 (Fig. 6). The apparent lack of association between the mean and the total fertility rate in the regional schedules for 1985-87 is repeated for the this time series of national schedules. Evidently, one must conclude that recent Swedish period fertility is characterized by a complete lack of association between the central age of childbearing and the level of reproduction.

Fig. 7 shows the changes in the mean age of the national fertility schedules between 1950 and 1986. One will notice that the mean age has declined sharply until about 1975 when it began to increase dramatically. This increase in the mean age of the fertility schedule is seen as a result of the deferment of births (see e.g. Lundström and Springfeldt, 1988, pp. 3-5). As the biological clock runs out for the large cohorts of women born during the post war period, there is an echo of these resulting in delayed births. However, superimposed on this effect may also be a *de facto* tendency for Swedish women to increase their fertility relative to the very low figures for the late 1970s. Fig. 8, for example, shows a clear tendency for the total fertility rate in the national schedules to undergo a current increase. And, as already mentioned, the most recent estimate for 1988 suggests a total fertility rate of nearly 1.95.

Notwithstanding the fact that there have been profound temporal variation in the level as well as in the age-pattern of Swedish period fertility, the changes in the level of reproduction for the cohorts since the 1930s have not changed very much. Indeed, the female cohorts born between 1930 and 1939 all have a total fertility rate (at age 45) of 2 children (see e.g. Qvist, 1987, p. 46) (rounding the figures to one decimal place, the exception is for the cohort of 1934 which reached 2.1 children at age 45). Changes in the age-pattern of fertility in Sweden are dealt with in further detail in Section 3.5. This Section also focuses on changes in the Australian age-pattern of fertility.



Fig. 2.-The skewness plotted against the total fertility rate for 284 municipalities in Sweden, 1985-87.



Fig. 3.-The skewness plotted against the mean for 284 municipalities in Sweden, 1985-87.



Fig. 4.-par2 plotted against the variance for 284 municipalities in Sweden, 1985-87



Fig. 5.-par2 plotted against the mean for 284 municipalities in Sweden, 1985-87



Fig. 6.-The mean plotted against the total fertility rate for Swedish national fertility schedules between 1950 and 1986



Fig. 7.-The mean plotted against time for Swedish national fertility schedules between 1950 and 1986



Fig. 8.-The changes in the total fertility rate in Swedish national fertility schedules between 1950 and 1986

3.5 The measures for a time series of Australian fertility schedules between 1929 and 1970

The correlations in Table 5, on the whole, are rather small. This is probably because the fertility levels in the Swedish municipalities are hovering above and below the level of replacement. As such, the measures do not correlate to the same extent as they might for a time series of fertility schedules. Although this is a conjectural point of view, it is supported e.g. by a time series of Australian fertility schedules for the 42 annual periods between 1929 and 1970. The principal reason for the inclusion of these schedules in the present paper is that they were easily accessible in Pollard, Yusuf and Pollard (1981, pp. 88-89).

For the Australian time series we find that the total fertility rate correlates highly with the mean (r = -0.7799) (Table 6). Therefore, the lower the mean, the higher the level of fertility. The skewness and the total fertility rate are also highly correlated (r = 0.8636).

One is tempted to interpret these results to the effect that a low mean age implies the possibility for women to increment their reproduction in terms of second, third and higher order births. More specifically, the manifestation of high fertility during the early periods of marriage, or cohabitation, means that the main bulk of first order births takes place while the women are young. The high skewness implies a tendency for women to replenish their fertility in terms of higher order births at a relatively late stage in life.

It is, however, somewhat unsatisfactory to argue in this fashion because whereas birth order statistics import precise knowledge concerning these matters, fertility schedules which mix all birth orders do not. Moreover, because the data we work with are period data, cohort reasoning is error prone because, after all, the synthetic cohort given by the 35 mixed cohorts is very unlikely to have the fertility of any real cohort.

Perhaps the greatest value of Table 6 is that it shows an empirical mode of <u>temporal</u> dynamics in basic fertility measures. Table 7 gives measures for the Australian fertility schedules similar to those in Table 4.

			ست منه ماه سب میں بہے ہے۔ اس میہ اس م			
	R	Mean	Variance	Skewness	par1	par2
R	1.0000	-0.7799	-0.7551	0.8636	-0.1169	0.4251
Mean		1.0000	0.9211	-0.9670	-0.3576	-0.7148
Variance			1.0000	-0.8939	-0.1109	-0.3943
Skewness				1.0000	0.1625	0.6556
par1					1.0000	0.7156
par2						1.0000

Table 6.-Correlations between fertility measures for 42 Australian schedules between 1929 and 1970

Table 7.-Fertility measures for 42 annual Australian schedules, 1929-70

Measure	Minimum	Mean	Maximum
R	2.11	2.83	3.56
Mean	27.09	2.05	29.33
Variance	31.49	35.92	42.00
Skewness	0.3310	0.4688	42.00 0.5960
par1	0.0700	0.0888	0.1180
par2	0.3380	0.3758	0.3980
-			

Fig. 9 shows how the total fertility rate varied between the years 1929 and 1970. Fig. 10 shows a plot of the mean against the total fertility rate. It will be noted that there is a clear negative correlation between the two measures. Fig. 11 shows par1 plotted against the mean age. As in the case of the Swedish regional schedules, the lack of association between par1 and the mean is striking in view of the general belief that par1 is closely related to the mean (see e.g. Brass et al. 1968, pp. 105-122). Fig. 12 shows par2 plotted against the mean. Here the association is much higher; however, even so, the data do not suggest that a regression representation for estimation of the mean from a known value of par2 would be successful.

Finally, in Fig. 13 we have shown the association between the total fertility rate and the mean for a selection of 55 empirical fertility schedules of which a large number come from developing countries (Hartmann, 1982, pp. 43-46). Here the association between the mean and the total fertility rate is the reverse of that in the Swedish and Australian schedules. (To illustrate this the regression line for regressing the total fertility rate on the mean is also shown.) The main reason for including this figure is to demonstrate that the relationship between the level and central location of fertility much depends on whether one considers a time series of national schedules, a selection of regional schedules or a selection of schedules from different countries and time periods. Indeed, there is not a more or less unique relationship between levels and shapes of fertility curves.

There are several reasons why one should expect to find extensive variation in the agepattern of fertility. In developing countries, for example, improvements in mortality may reduce the risk of mortality due to child birth. There are situations where this could bring about a positive correlation between the mean age and the total fertility rate. On the other hand, the implementation of modern family planning practices may primarily be directed to older women with the effect that higher birth order births are averted whereas there is little or no prevention of first order births. This may result in a negative, or at least negligible, correlation between the mean and the total fertility rate. Then again, if delayed childbearing is considerable then this may impose a positive correlation between the mean and the total fertility rate. Therefore, it is not generally true that the age at first birth is "closely" related to the level of completed fertility. This is well illustrated not only by the Swedish regional data for 1985-87 but also by the national schedules since 1950. In the case of the Australian data, however, there is a clear case for arguing that such a relationship holds.



Fig. 9.-The total fertility rate in Australia for the years between 1929 and 1970



Fig. 10.-The mean plotted against the total fertility rate for Australia, 1929-1970



Fig. 11.-par1 plotted against the mean for Australia, 1929-1970



Mean age of fertility schedule

Fig. 12.-par2 plotted against the mean for Australia, 1929-1970



Mean age of fertility schedule

Fig. 13.-The total fertility rate plotted against the mean for a selection of 55 empirical fertility schedules



Fig. 14.-The variance plotted against time for the Australian schedules, 1929-1970



Fig. 15.-The variance plotted against time for the Swedish schedules, 1959-1986



Fig. 16 Skewness plotted against time for the Australian schedules, 1929-1970



Fig. 17.-Skewness plotted against time for the Swedish schedules, 1950-1986

In terms of changes in the age-pattern of fertility, one will notice that over time there has been a marked decline in the variance of the fertility schedule. Such trends can be observed for both Australia and Sweden (Figs. 14 and 15). (No doubt, they can be observed for most, if not all, industrialized societies). The decline in the variance over time means that women produce their children within a shorter span of years than previously. However, it does not necessarily mean that they produce fewer children; as already noted, in the case of Sweden changes in cohort fertility since the 1930s have been rather minor.

Changes over time in the skewness can also be observed. For the Australian schedules, the skewness increased at a time when the total fertility rate increased (Fig. 16). For the Swedish schedules, the skewness has fallen together with a falling total fertility rate (Fig. 17). Consequently, both sets of schedules confirm a strong positive correlation between the total fertility rate and the skewness. In the case of Sweden, the skewness is now quite small. This will also appear in Section 5 where we show a number the graphs of a number of county fertility schedules.

In terms of regional variation in fertility, Tables 4 and 7 clearly show that there is considerable variation in the age-pattern in Sweden. In Sweden around 1986, the skewness ranged from about zero to 2.3. In Australia between 1929 and 1970, the skewness only ranged between 0.3 and 0.6. In the Swedish schedules between 1950 and 1986 (Fig. 17), the skewness ranged between about 0.45 and 0.95.

4. GRADUATION AND REPRESENTATION OF FERTILITY CURVES

4.1 Basic considerations concerning the graduation of a fertility curve

In broad demographic contexts, the notion of graduation is often used so that the reader is left with a vague understanding of what is meant. Sometimes graduation refers to the mere process of smoothing a curve. On occasion, it refers not only to curve fitting but also to testing whether the fitted entities are statistically commensurate with the observed ones. Smoothing, generally speaking, is understood either to be done in terms of hand-fitting (which may be seen as the simplest approach) or in terms of more sophisticated means. In actuarial circles, graduation usually has a more precise meaning. For example, Haycocks and Perks (1955, p. 9) write, albeit in the context of mortality, that:

The object of the process of graduation is to smooth the progression of the rates from age to age. That is the practical aim. The theoretical basis is the assumption that each rate contains a random error and that, shorn of these errors, the rates would show a smooth progression from age to age. Smoothness is a concept that has eluded a precise and generally accepted mathematical definition, but for actuarial purposes it usually means that the successive differences of the function concerned diminish and that third differences are small [in actuarial science, second and third order polynomials, at least in older literature, are referred to as smooth curves].

It follows as a logical consequence, that it is a technique that may, but not necessarily should, be employed when variation in the observed rates is clearly visible. If one has reason to believe that the fluctuations in the observed rates are solely due to the effects of randomness, then it is justified to estimate the true underlying rates by means of graduation. In that case, however, it is essential to test if the graduated rates are commensurate, in a statistical sense, with the observed ones. Consequently, graduation is a process which entails a statistical test of whether the graduated rates yield graduated events (births or deaths as the case may be) which are in statistical agreement with the observed ones.

In situations where the population at risk is large, the variance in the age-specific birth rates, say, may be virtually nil so that even a model curve that nearly coincides with the observed one leads to a rejection of the test that observed and graduated births are drawn from the same parent population. In that case, it does not follow that one should dismiss the outcome of the smoothing experiment because, after all, it may be of interest in its own right that the model curve displays the same overall feature as the observed one. To avoid confusion, however, one should <u>not</u> refer to the model curve as a graduating curve; for the exercise is <u>not</u> one of graduation, but one of representing the observed curve by means of a model curve. The model curve may derive from spline fitting, polynomial fitting, hand-drawing, fitting a parametric function, or from some other procedure. The heart of the matter is not how the model curve is constructed but the conceptual purpose it serves.

In this paper we see the process of graduation as one that applies to fertility schedules which display visible random variation. Now, it is not by any means obvious that all the variation displayed by the rates stem from the effects of randomness. But, in the vast majority of cases, it is simply not possible to determine which other factors operate on the rates. It is a convenient assumption, then, to assume that all variation is due to randomness.

As one proceeds from small populations of women to larger ones, there is a clear tendency for the rates to display a decreasing amount of randomness. The questions that springs to mind is when one should see the rates as smooth in their own right. Fig 11 illustrates this situation. The figure shows the plots of the fertility rates against age for Sweden (4,252,741 women), the county of Stockholm (829,020 women) and the municipality of Arjeplog (1,808 women).

The curve for Sweden is smooth. The curve for the county of Stockholm displays some minor ragged features. This raggedness is not necessarily attributable to the effects of random

variation. Whether one should apply graduation to such a curve is a moot point. In fact, whether the observed or a set of nicely smoothed rates should be used when projecting the population of the county of Stockholm, for all practical purposes, must be a matter of splitting hairs. With respect to the municipality of Arjeplog, the situation is different. Here we see the typical random features of a fertility curve derived from a small population of women. Here the purpose of graduation is to estimate the true underlying rates. More specifically, if we imagine that the random mechanism which generated the observed schedule for Arjeplog were to operate a large number of times, then, by taking an average of a large number of schedules, we would eventually arrive at an average schedule which is perfectly smooth. It is the purpose of graduation to capture this imaginary average schedule which gives the true underlying rates.



Fig. 18.-Normalized fertility schedules for Sweden, the county of Stockholm and the municipality of Arjeplog.

In further support of the discussion, we quote Hoem and Berge (1974, p. 363) who write:

"The diagram of age-specific fertility rates for a population, based on data for a calendar period, say, will typically picture a curve which looks much like a left-skewed unimodal probability density, such as a gamma density or some of the beta densities (starting just below age 15), for instance but with superposed fluctuations. Unless the population is very large, the diagram of the sequence of fertility rates, plotted against age, will have quite a ragged appearance [see Fig. 18 above]. It is frequently assumed that "real fertility" would be portrayed by a smooth curve and that the irregularities of the observed curves are due to accidental circumstances. The observed fertility rates are then regarded as "raw" or primary estimates of the underlying "real" rates, and graduation is employed to get a smoother curve."

Graduating the rates, then, is an exercise which should be seen partly in the perspective of how large the underlying population of women is, partly in the perspective of the particular purpose that the graduated rates are meant to serve. It does not follow from these considerations, however, that it is meaningless to fit a parametric model to the national rates. For example, if this exercise is meant to show that the national rates, in an impressionistic sense, are closely modeled by a parametric curve, then this might be of considerable interest in its own right. Furthermore, in the context of making population projections, a parametric representation of the fertility curve, even if it introduces a significant bias, may be an aid in understanding the extent to which different shapes of the fertility curve may lead to differences in the projected figures (subject to a fixed total fertility rate). (Putting it differently, it is easier to manipulate the shape of a parametric curve, by varying the parameters, than it is to manipulate the shape of an observed curve.)

As previously noted, the demographic literature has a somewhat obtuse attitude towards the purpose of graduation. For instance, in the now classic **Methods and Materials of Demography** (Shryock and Siegel, 1971, p. 813), it is noted that methods of graduation, interpolation and extrapolation (based on mathematical functions) often modify the raw measures greatly, substituting for them an idealized model of reality. It is also noted that graduation techniques only are justified if the basic data are essentially reliable and the analyst's task is merely to remove the effects of random deviations from the true underlying values.

The problem with this outline is that, in our view, it does not distinguish clearly between two important cases, namely (1) that the rates derive from a large population and display a smooth curve, and (2) that the rates derive from a small population and display typical features of randomness. In the first instance, a fitted curve should be referred to as a representation of the observed curve, in the second instance, the fitted curve, depending on how well it fits the observed curve, could be referred to as a graduated curve. To illustrate this point, we quote Hoem et al. (1981, p. 231) who, when modeling Danish fertility curves write:

"We confine our interest to the use of a parametric function to provide an accurate, smooth, and parsimonious **representation** of a known set of fertility rates for single-year age groups in a situation where random variation in the "raw" rates can be disregarded."

We now turn to a discussion of parametric functions used to represent or graduate fertility curves.

4.2 Statistical attempts to model the fertility curve

Statistical attempts to model the fertility schedule by means of parametric functions can be traced back to the 1930s when Wicksell (1931) and Lotka (1939) graduated the net maternity function by means of probability density functions. Wicksell (1931) showed in an often quoted paper that the gamma density can be used as a model of the typical age-pattern of fertility. Lotka preferred to work with the normal density function (see e.g. Keyfitz, 1968, pp. 140-143), and Hadwiger (1940) and Hadwiger and Ruchti (1941) presented and demonstrated an ingenious parametric function for graduation of the net maternity function (see also Gilje, 1969, p. 118; and Keyfitz, 1968, pp. 140-169).

It is not amiss to note that there is an important conceptual difference between the models proposed by Coale and Trussell (1974) and those proposed by Wicksell, Lotka and Hadwiger. While Wicksell, Lotka and Hadwiger focused on graduation of the net maternity function, the approach taken by Coale and Trussell was one of identifying typical age-patterns of human fertility as well as to provide a measure of the degree of fertility control. In addition, it should be noted that the main spheres of application probably were intended to be in reconstruction of fertility schedules from historical records and in indirect estimation of fertility and child mortality in developing countries.

Although the model fertility schedules proposed by Coale and Trussell must be deemed very successful, we have chosen not to elaborate with them in the present context. The basic

reason for this is that we wish to show that the much simpler gamma probability density function provides a parsimonious description of Swedish fertility (and, presumably, of all low fertility schedules).

4.3 The gamma probability density function as a model of fertility

The gamma probability density function

$$g(x; c, k, d) = \frac{c^k}{\Gamma(k)} (x - d)^{k-1} \exp(-c(x-d)), x \ge d$$
(13)

with

$$\Gamma(k)\approx \sqrt{\frac{2\pi}{k}} \; k^k \exp(-k+1/(12k))$$

gives a model of the normalized fertility schedule $\{f_X/R\}$.

The mean and variance of (13) are $\mu = d + k/c$ and $\sigma^2 = k/c^2$, respectively. The i:th central moment for the gamma distribution is $\mu_i = k(k+1)...(k+i-1)/c^i$. The skewness is $\gamma = \mu_3/\sigma^3 = 2/\sqrt{k}$.

To model an empirical schedule $\{f_x\}$, one lets

$$h(x; R, c, k, d) = R g(x; c, k, d)$$
(14)

and estimate θ = (R, c, k, d) by minimizing

$$\sum_{\mathbf{x}} (\mathbf{f}_{\mathbf{x}} - \mathbf{h}(\mathbf{x}; \boldsymbol{\theta}))^2 \tag{15}$$

with respect to θ . In the present study, the non-linear module in (Macintosh) SYSTAT has been used to minimize (15). The starting values for minimization of (15) are the moment estimates R, $c = \mu/\sigma^2$ and $k = \mu^2/\sigma^2$. The starting value of d was usually taken to be 10 years. Denoting the estimated parameters by $\hat{\theta}$, the fitted rates are obtained from (14) by letting

$$f_{x}^{*} = h(x; \hat{\theta}).$$

From a theoretical point of view, it should be noted that whereas g(x; c, k, d) is a probability density function, the observed rates f_x are intensities, and $f_x = h(x; \hat{\theta})$ gives their model representation. The raw rate $f_x = B_x/E_x$ may be considered asymptotically normally distributed with estimated mean B_x/E_x and estimated asymptotic variance f_x/E_x . In addition, the f_x may be considered asymptotically independent (see e.g. Hoem and Berge, 1974, p. 365).

Consequently, and assuming that f_x is not zero,

$$Q = \sum_{\mathbf{X}} [f_{\mathbf{X}} - \mathbf{h}(\mathbf{x}; \hat{\boldsymbol{\theta}})]^2 E_{\mathbf{X}} / f_{\mathbf{X}}$$
(16)

follows approximately a Chi-square distribution with N - j degrees of freedom where N is the number of ages and j is the number of parameters (here 4). If the schedule covers the age range 14 to 49 years, (16) has 36 - 4 = 32 degrees of freedom. (As noted, in situations where f_X is zero for some of the ages, these are deleted in the calculation of Q.)

4.4 An application to the municipality of Dals-Ed

Dals-Ed is a small community with a total female population of 2,641 (as of January 1, 1988) in the county of Älvsborg. The total fertility rate for the three-year period 1985-87 is R = 1.81 (see Table A1). Because of the small population of women, it is evident that the age-specific fertility rates display large variation and may call for graduation if, for example, they were to be used in a projection of the future population of Dals-Ed. Table 8 gives the observed rates for Dals-Ed. Using these rates for estimation of R, c, k, and d, minimization of (15) over the ages 14.5 to 49.5 yields $\hat{R} = 1.8022$, $\hat{c} = 0.4816$, $\hat{k} = 6.8169$ and $\hat{d} = 14.5$. The observed and fitted rates together with the individual contributions for Q appear in Table 8. Fig. 19 shows the observed and fitted rates . In an impressionistic sense, it would appear that the fit is acceptable. This is also confirmed by testing if the observed and fitted rates are commensurate with the same underlying experience.

The test variable Q follows approximately a Chi-square distribution with N' - 4 degrees of freedom where N' is the number of ages for which the observed rate is different from zero, that is, we exclude the ages for which the recorded births are nil. Here N' = 26 (Table 8) so that the number of degrees of freedom is 22. The result of the test variable Q = 13.97 which is well below the 5 percent limit of about 33.9.



Fig. 19.-Observed and fitted rates for the municipality of Dals-Ed

14.5 0.000 0.000 114 15.5 0.000 0.000 110 16.5 0.009 0.001 117 0.832 17.5 0.008 0.003 123 0.384 18.5 0.017 0.011 116 0.245 19.5 0.000 0.025 113 20.5 0.041 0.045 97 0.037 21.5 0.038 0.068 105 2.486 22.5 0.100 0.092 100 0.644 23.5 0.140 0.113 100 0.520 24.5 0.146 0.128 89 0.197 25.5 0.126 0.138 87 0.099 26.5 0.158 0.142 76 0.123 27.5 0.131 0.132 87 0.244 29.5 0.126 0.132 87 0.247 33.5 0.166 0.097 89 1.195 32.5 0.069		Fertility	v rate	Female	
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22.50.1000.0921000.064 23.5 0.1400.1131000.520 24.5 0.1460.128890.197 25.5 0.1260.138870.099 26.5 0.1580.142760.123 27.5 0.1310.139840.041 28.5 0.1260.132870.024 29.5 0.1210.122910.000 30.5 0.0920.110870.306 31.5 0.0670.097891.195 32.5 0.0690.083870.247 33.5 0.1160.070861.568 34.5 0.0570.059880.006 35.5 0.0430.048930.054 36.5 0.0290.0391020.351 37.5 0.0830.031973.160 38.5 0.0290.0251030.056 39.5 0.0180.0121260.2524 41.5 0.0000.005920.004 41.5 0.0000.005920.004 41.5 0.0000.003780.44 45.5 0.0000.002790.44 45.5 0.0000.002790.44 45.5 0.0000.002790.44	20.5	0.041	0.045	97	0.0379
23.5 0.140 0.113 100 0.520 24.5 0.146 0.128 89 0.197 25.5 0.126 0.138 87 0.099 26.5 0.158 0.142 76 0.123 27.5 0.131 0.139 84 0.041 28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.005 92 0.007 102 41.5 0.000 0.003 78 0.000 0.002 79 48.5 0.000 0.002 79 0.002 79 0.002	21.5	0.038	0.068	105	2.4868
24.5 0.146 0.128 89 0.197 25.5 0.126 0.138 87 0.099 26.5 0.158 0.142 76 0.123 27.5 0.131 0.139 84 0.041 28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.067 0.097 89 1.195 32.5 0.067 0.097 88 0.006 31.5 0.067 0.097 88 0.006 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.554 36.5 0.029 0.039 102 0.351 37.5 0.083 0.012 126 0.2524 40.5 0.000 0.015 126 0.2524 41.5 0.008 0.012 126 0.2524 42.5 0.009 0.007 102 1.7024 44.5 0.000 0.005 92 0.007 0.004 45.5 0.000 0.002 79 0.002 79 48.5 0.000 0.002 79 0.000 0.002	22.5	0.100	0.092	100	0.0640
25.5 0.126 0.138 87 0.099 26.5 0.158 0.142 76 0.123 27.5 0.131 0.139 84 0.041 28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.54 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.005 92 0.025 41.5 0.000 0.005 92 0.007 0.001 43.5 0.000 0.002 79 0.000 45.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	23.5	0.140	0.113	100	0.5207
26.5 0.158 0.142 76 0.123 27.5 0.131 0.139 84 0.041 28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351^{11} 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.0061 40.5 0.000 0.005 92 0.007 41.5 0.000 0.005 92 0.000 45.5 0.000 0.003 78 0.000 45.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	24.5	0.146	0.128	89	0.1975
27.5 0.131 0.139 84 0.041 28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 41.5 0.000 0.005 92 42.5 0.000 41.5 0.000 0.005 92 44.5 0.000 0.002 45.5 0.000 0.003 78 46.5 0.000 0.002 79 48.5	25.5	0.126	0.138	87	0.0994
28.5 0.126 0.132 87 0.024 29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.005 92 0.007 41.5 0.000 0.005 92 0.000 44.5 0.000 0.003 78 0.000 46.5 0.000 0.002 79 0.002	26.5	0.158	0.142	76	0.1231
29.5 0.121 0.122 91 0.000 30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 41.5 0.000 0.015 126 1.702 42.5 0.009 0.007 102 1.702 44.5 0.000 0.005 92 1.702 45.5 0.000 0.003 78 1.75 48.5 0.000 0.002 79 1.792	27.5	0.131	0.139	84	0.0410
30.5 0.092 0.110 87 0.306 31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 41.5 0.008 0.012 126 0.2524 42.5 0.009 0.007 102 1.7023 43.5 0.000 0.005 92 0.000 43.5 0.000 0.003 78 0.000 45.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	28.5	0.126	0.132	87	0.0249
31.5 0.067 0.097 89 1.195 32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 41.5 41.5 0.008 0.012 126 0.2524 42.5 0.009 0.007 102 1.7023 44.5 0.000 0.005 92 0.007 45.5 0.000 0.003 78 0.000 47.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	29.5	0.121	0.122	91	0.0008
32.5 0.069 0.083 87 0.247 33.5 0.116 0.070 86 1.568 34.5 0.057 0.059 88 0.006 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 0.2526 41.5 0.008 0.012 126 0.2526 42.5 0.009 0.009 115 0.000 43.5 0.000 0.005 92 0.007 45.5 0.000 0.003 78 0.000 45.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	30.5	0.092	0.110	87	0.3064
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	31.5	0.067	0.097	89	1.1955
34.5 0.057 0.059 88 0.066 35.5 0.043 0.048 93 0.054 36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 41.5 0.008 0.012 126 0.2524 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.7023 44.5 0.000 0.005 92 0.007 45.5 0.000 0.003 78 0.000 47.5 0.000 0.002 79 0.002 48.5 0.000 0.002 79 0.002	32.5	0.069	0.083	87	0.2471
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	33.5	0.116	0.070	86	1.5688
36.5 0.029 0.039 102 0.351 37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 0.252 41.5 0.008 0.012 126 0.252 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 0.000 45.5 0.000 0.003 78 0.000 46.5 0.000 0.002 79 0.002	34.5	0.057	0.059	88	0.0062
37.5 0.083 0.031 97 3.160 38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 126 41.5 0.008 0.012 126 0.252 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 145 45.5 0.000 0.003 78 145 46.5 0.000 0.002 79 148.5	35.5	0.043	0.048	93	0.0541
38.5 0.029 0.025 103 0.056 39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 126 41.5 0.008 0.012 126 0.252 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 14.5 45.5 0.000 0.003 78 14.5 46.5 0.000 0.002 79 14.5	36.5	0.029	0.039	102	0.3517
39.5 0.018 0.019 111 0.006 40.5 0.000 0.015 126 126 41.5 0.008 0.012 126 0.252 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 145 45.5 0.000 0.003 78 145 46.5 0.000 0.002 79 148.5	37.5	0.083	0.031	97	3.1601
40.5 0.000 0.015 126 41.5 0.008 0.012 126 0.2520 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.7022 44.5 0.000 0.005 92 92 45.5 0.000 0.003 78 94 46.5 0.000 0.002 79 94 48.5 0.000 0.002 79 94	38.5	0.029	0.025	103	0.0568
41.5 0.008 0.012 126 0.2524 42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 92 45.5 0.000 0.003 78 94 46.5 0.000 0.002 79 94 48.5 0.000 0.002 79 94	39.5	0.018	0.019	111	0.0062
42.5 0.009 0.009 115 0.000 43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 1 45.5 0.000 0.004 78 1 46.5 0.000 0.003 78 1 47.5 0.000 0.002 79 1	40.5	0.000	0.015	126	-
43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 1.702 45.5 0.000 0.004 78 1.702 46.5 0.000 0.003 78 1.702 47.5 0.000 0.002 79 1.702 48.5 0.000 0.002 79 1.702	4 1.5	0.008	0.012	126	0.2520
43.5 0.029 0.007 102 1.702 44.5 0.000 0.005 92 102 1.702 45.5 0.000 0.004 78 102 1.702 46.5 0.000 0.003 78 102 1.702 47.5 0.000 0.002 79 102 1.702 48.5 0.000 0.002 79 102 1.702	42.5	0.009	0.009	115	0.0000
44.5 0.000 0.005 92 45.5 0.000 0.004 78 46.5 0.000 0.003 78 47.5 0.000 0.002 79 48.5 0.000 0.002 79	43.5	0.029	0.007	102	1.7023
46.5 0.000 0.003 78 47.5 0.000 0.002 79 48.5 0.000 0.002 79	14.5	0.000	0.005	92	-
46.5 0.000 0.003 78 . 47.5 0.000 0.002 79 . 48.5 0.000 0.002 79 .	5.5	0.000			-
17.5 0.000 0.002 79 - 18.5 0.000 0.002 79 -	6.5	0.000			-
l8.5 0.000 0.002 79 -	7.5	0.000			-
	18.5	0.000			-
19.5 0.000 0.001 /9	19.5	0.000	0.001	79	-

Table 8.- Observed and fitted age-specific fertility rates for the municipality of Dals-Ed, 1985-87.

Sum of squares Q

13.97

5.0 MODELING SWEDISH PERIOD FERTILITY AROUND 1986

5.1 Modeling the national fertility schedule

In this section we show that the gamma probability density function gives close fits to the Swedish regional fertility experiences around 1986. As we have already noted, there are situations where one must carefully distinguish between graduating or modeling the observed fertility curve. Because the counties have fairly large population sizes and, in addition, the age-specific fertility rates are estimated from births taking place over a three-year period, the observed curves, with the exception of that of Gotland (Fig. 25), are almost smooth. Hence, fitting (14) to the observed rates is not an exercise in graduation but one of showing that, in an impressionistic sense, the chosen model curve gives an adequate fit to the observed rates.

Although there is some latitude for discussing what is a "good" fit, one must bear in mind that in the case of large populations, there is no generally accepted standard for what is, or is not, a close fit. Actually, the most common way of inspecting the goodness of fit appears to be by eye balling the diagram of observed and fitted rates. There is nothing improper about such a procedure; it is simple and expedient, and it leads to a fast decision as to whether one should accept the fitted rates as sufficiently accurate.

The observed and fitted curves for Sweden are given in Fig. 20. The births, mid-year populations, observed and fitted fertility rates are given in Table A3. The fitted curve has estimated parameters $\hat{R} = 1.8101$, $\hat{c} = 0.8596$, $\hat{k} = 20.1210$ and $\hat{d} = 5.5854$. Certainly, it would appear that the fitted curve captures the essential characteristics of the observed one. However, a careful inspection of the curve reveals that the fitted rates for ages above 40 years tend to be too high. This, regrettably, is a general feature of using the gamma probability density function as a model of fertility. Nevertheless, for practical purposes, the bias in the fitted rates over age 40 is of little or no importance. (In fact, it would be the feature of a good model that it would fail in providing a close fit to the rates of the least interest, namely the rates for women approaching the end of their reproductive ages.) With respect to the remaining part of the curve, the fit is quite close (Fig. 16). Notice, however, that for very young women, the fitted rates do not coincide as nicely as for women between 20 and 40 years, say. Then again, the rates for women below 20 are usually quite small so that even if the fit is rough, it remains adequate for most practical applications.

In modeling the fertility curve for the county of Stockholm (Fig. 21), the general failure of the gamma probability density function to capture the age-pattern of the rates for women above age 40 is clearly brought out. However, as we have already noted, this failure is not serious when we bear in mind that both the observed and fitted rates are very small. Incidentally, and in comparison with all the regional fits, Fig. 21 displays an unusually poor fit. Hence, in the context of using the gamma probability density function as a model of Swedish regional fertility, the fit to the schedule for the county of Stockholm provides a lower limit of the ability of this function to portray the age-pattern of fertility.

In the case of modeling the fertility curve for the county of Uppsala (Fig. 22), the *tour de force* of the gamma probability function is brought out in full. Here, apart from the usual failure of giving a close fit to the rates for women aged 40 and over, the fit is most satisfactory.

Fig. 23-27 show additional fits to the county schedules for Kronoberg, Blekinge, Gotland, Halland, and Norrbotten.





Fig. 20.-Observed and fitted age-specific fertility rates for Sweden 1985-87



Stockholm

Fig. 21.-Observed and fitted rates for the county of Stockholm, 1985-87


Fig. 22.-Observed and fitted rates for the county of Uppsala, 1985-87



Kronoberg

Fig. 23.-Observed and fitted rates for the county of Kronoberg, 1985-87

Blekinge



Fig. 24.-Observed and fitted rates for the county of Blekinge, 1985-87



Gotland

Fig. 25.-Observed and fitted rates for the county of Gotland, 1985-87





Fig. 26.-Observed and fitted rates for the county of Halland, 1985-87



Fig. 27.-Observed and fitted rates for the county of Norrbotten, 1985-87

Based on Figs. 20-27, it would appear that the gamma probability density function provides an accurate model of current Swedish period fertility. In fact, over the years, (14) has become an increasingly realistic model of the fertility schedule. Table A4 gives

$$S = \sum [f_x - f_x^*]^2,$$

where f_x is the observed and f_x^* is the fitted rate, for each of the 37 annual fertility

schedules for Sweden between 1950 and 1986². These schedules are given by five-year age groups of women. An inspection of Table A4, which gives the estimated parameters for each of the schedules as well as the corresponding sum of squared deviations, indicates that the sum of squared deviations has decreased dramatically over time; hence, over time, (14) has become an increasingly realistic model of Swedish fertility (Fig. 28). (Notice that S for each of the Swedish schedules between 1950 and 1986 is a sum of 7 squared deviations because we used the schedules given by five-year age groups of women). Figs. 29 and 30 illustrate the improvements in the goodness of fit between 1955 and 1964. (From 1964 and onwards, the fitted and observed rates virtually fall on top of one another in the diagrams.)

With respect to the Australian schedules, a similar finding applies (Fig. 31). Table A5 gives the estimated parameters for each of the Australian schedules as well as the sum of squared deviations (now based on 35 squared deviations since the schedules are given by single-year ages of women). Around 1940, after a long decline in the level of Australian fertility, the goodness of fit improved but as fertility rose during 1940-60, the goodness of fit decreased. However, after 1960, the age-pattern in Australia has come close and closer to that determined by (14).

There is reason to expect, then, that in a general perspective, the gamma probability density function is a particularly reliable model of fertility when fertility is close to the level of replacement. Incidentally, in modeling Danish fertility curves for 1962 to 1971 Hoem et al (1981, pp. 231-244) also reach the conclusion that the gamma probability density function fares extremely well in comparison with other models e.g. the Hadwiger model and the model of Coale and Trussell (see in particular Hoem et al. 1981, p. 234).

²

Table A4 gives 10⁶ times the sum of squared deviations



Fig. 28.-Goodness of fit for Swedish national fertility schedules between 1950 and 1986



Fig. 29.-Observed and fitted age-specific fertility rates for Sweden, 1955



Fig. 30.-Observed and fitted age-specific fertility rates for Sweden, 1964



Fig. 31.-Goodness of fit for Australian fertility schedules between 1929 and 1970

5.2 Interpreting the parameters in the gamma probability density function as parameters of fertility

It should be noted that in this paper the parameters R, c, k and d in (14) have been estimated jointly by means of the non-linear module in Macintosh Systat. Hence, we have not retained the observed value of R as the final estimate of R. Table 9, which gives the estimated parameters along with the raw moments R, μ and σ^2 , shows that there is a slight difference between R and \hat{R} . This difference should not be misconstrued in the sense that it is interpreted as an error. Both R and \hat{R} are estimates of the total fertility rate. R is a moment estimate while \hat{R} derives from the minimization of (15). Likewise there is a small difference between the mean and variances in the observed and fitted curves (see e.g. Hoem and Holmbeck, 1975).

It stands to reason that one would like to interpret the parameters in (14) in terms of basic fertility characteristics. Unfortunately this is not so easy. For example it is tempting to interpret d as a measure of the start of childbearing (and this is often the way it is interpreted). An inspection of Table 9 reveals that such an interpretation is invalid. The same conclusion is drawn from inspecting Table A4 and Table A5 which give the estimated parameters for Sweden 1950-86 and Australia, 1929-70, respectively. Parameter d is, as we shall see, principally a measure of the skewness of the fertility curve.

Because the skewness of the model curve is $\gamma = 2/\sqrt{k}$, it is clear that k should be seen exclusively as a measure of the skewness of the fertility curve. It is not clear, however, how one should interpret c. This parameter correlates poorly with the mean and with other characteristics in the observed curve. Parameter d, on the other hand, is closely tied to the skewness of the curve. Figs. 32 and 33 show d plotted against c and k, respectively. In Fig. 33, especially, it becomes clear that there is an almost perfect linear relationship between k and d. Hence, d as well as k, are principally geared to the skewness of the curve. Because the skewness has decreased in both the Swedish and the Australian national schedules, the estimates of d over time have shown a decreasing tendency (this is illustrated in Fig. 37).

With respect to the parameters c and k they almost always hang together in a more or less linear manner. Fig. 34 shows a plot of k against c for the Swedish regional schedules. This parameter trace is typical of fertility. Fig. 35 shows the result of plotting k against c for the national Swedish schedules, 1950-86. Fig. 36 shows c plotted against time for the Swedish national fertility schedules between 1950 and 1986. It will be seen that c has changed in a relatively linear manner. The same applies to k. Hence, in some situations, it may be possible to project fertility trends by means of specifying c and k as functions of time.

Fig. 38 shows a plot of k against c for the Australian schedules. Here there is a great deal of fluctuation at the beginning of the series. However, towards the end of the period, there is a clear linear trend between c and k. Finally, in Fig 37 we have plotted estimates of d against time for the Swedish national schedules between 1950 and 1986.

Table 9.-Observed and estimated parameters in modeling the 25 country fertility schedules for Sweden, 1985-87

	Observed			Estimated			
County	R	μ	σ ²	Â	¢ c	ĥ k	â

Stockholms län	1.66	29.6	27.4	1.69	0.8288	22.0296	3.34
Uppsala län	1.83	29.2	25.7	1.86	0.8487	20.8585	4.80
Södermanlands län	1.84	28.3	23.4	1.85	0.8083	16.2736	8.11
Östergötlands län	1.86	28.7	24.3	1.87	0.7716	15.4610	8.68
Jönköpings län	1.98	28.7	22.7	1.98	0.6992	11.6913	11.95
Kronobergs län	1.91	28.8	22.7	1.92	0.8272	16.1488	9.38
Kalmar län	1.90	28.5	23.2	1.90	0.6826	11.0210	12.35
Gotlands län	1.84	28.7	24.0	1.85	0.5839	9.2751	13.05
Blekinge län	1.89	28.3	23.0	1.90	0.7276	13.1627	10.16
Kristianstads län	1.89	28.5	23.4	1.89	0.6795	11.2435	12.05
Malmöhus län	1.73	29.0	24.9	1.74	0.8836	20.9709	5.41
Hallands län	1.87	29.0	22.2	1.88	0.8597	16.8287	9.47
Göteborgs och Bohus län	1.71	29.3	25.7	1.74	0.8665	22.1139	4.04
Älvsborgs län	1.90	28.7	23.4	1.91	0.7490	14.0349	10.02
Skaraborgs län	1.95	28.5	23.3	1.96	0.6814	11.4462	11.73
Värmlands län	1.81	28.5	24.0	1.82	0.7667	15.0323	8.99
Örebro län	1.81	28.4	24.4	1.82	0.8129	17.4434	7.01
Västmanlands län	1.81	28.4	23.6	1.82	0.6720	11.5000	11.34
Kopparbergs län	1.95	28.7	24.4	1.96	0.6161	10.1171	12.41
Gävleborgs län	1.78	28.5	24.1	1.80	0.5799	9.1317	12.89
Västernorrlands län	1.82	28.6	22.9	1.83	0.7024	12.0202	11.55
Jämtlands län	1.91	29.0	25.4	1.94	0.8757	22.0980	3.93
Västerbottens län	1.92	29.2	23.7	1.93	0.6318	10.0953	13.31
Norrbottens län	1.83	28.7	24.8	1.83	0.6892	12.3469	10.87



Fig. 32.-Estimates of d plotted against estimates of c for 25 Swedish county fertility schedules, 1985-87



Fig. 33.-Estimates of d plotted against estimates of k for 25 Swedish county fertility schedules, 1985-87



Fig. 34.-Estimates of k plotted against estimates of c for 25 Swedish county fertility schedules, 1985-87



Fig. 35.-Estimates of k plotted against estimates of c for Swedish national fertility schedules between 1950 and 1986.



Fig. 36.-Estimates of c plotted against time for Swedish national fertility schedules between 1950 and 1986.



Fig. 37.-Estimates of d plotted against time for Swedish national fertility schedules between 1950 and 1986



Fig. 38.-Estimates of k plotted against estimates of c for the Australian schedules, 1929-1970

6. DISCUSSION

6.1 Variations in regional fertility

The fertility schedules for the 284 Swedish municipalities relating to the three-year period 1985-87 display considerable variation not only with respect to the level of fertility but also with respect to age-patterns. The total fertility rate varies between 2.6 and 1.5. In terms of variation in the age-pattern of fertility, it is especially the skewness that varies across municipalities.

At the county level, there is not much variation in the level of fertility. The total fertility rate varies between 2.0 and 1.7. Hence, at the moment, all the county schedules result in inadequate reproduction. It may well be, however, that in the near future national Swedish period fertility recovers to the level of reproduction. In recent years, the total fertility rate has increased markedly and is about 1.95 for the first half of 1988.

The variation in the age-pattern of fertility is quite considerable. This means that although national fertility is about 15 percent below the level of reproduction, this does not imply a more or less constant regional age-pattern of fertility. Finally, it should be noted that, over time, there has been a remarkable decline in the variance of the national fertility curves. This is illustrated by both the Australian and Swedish time series of fertility schedules. In other words, in the two time series, the national schedules have become increasingly narrow - and display relatively little skewness (although the skewness varies considerably over the Swedish municipalities). Hence, the compensatory effect of delayed childbearing does not show up in the current national schedule for Sweden. However, as already noted, period schedules which mix all birth orders do not facilitate a detailed analysis of delayed childbearing.

6.2 The gamma probability density function as a model of fertility

The paper demonstrates that the gamma probability density function appears to be a useful model of current Swedish fertility. This is not a new finding in as much as **Statistics Sweden** already has used it for smoothing the municipal age-specific fertility rates. The smoothed rates, however, are only available in printouts from the Population Division in Örebro. But, whereas the present paper uses the four-parameter specification (14) and estimates the parameters jointly by means of the non-linear module in (Macintosh) Systat, **Statistics Sweden** has made use of a somewhat simpler procedure whereby parameter d is fixed and the estimate of the total fertility rate is the raw estimate R. Although we have not evaluated the differences in goodness of fit between these two approaches, it is fairly obvious that (14) on account of its additional parameter d and the refined estimation procedure used in the present study gives a closer fit than the simpler method used by the Population Division in Örebro.

The paper shows that over time the gamma probability density function has become an increasingly accurate model of fertility. A time series of Australian as well as Swedish schedules have been used to demonstrate this important finding. Again, the finding that the gamma probability density function gives a close fit to low fertility is not new since Hoem et al (1981) show that this function gives close fits to recent Danish fertility curves. Furthermore, it is a model which has deep roots in actuarial/demographic work because of its well-known ability do portray observed fertility curves. The new elements in the present paper are that (1) over time, there has been a remarkable improvement in the fits and (2) the estimation of the parameters is not carried out by the common method of moments but have been estimated jointly by means of minimizing (15). In addition the paper outlines when one should refer to the smoothed rates as graduated and when one should prefer to see the fitted rates as model rates which, in an impressionistic sense, portray the observed curve well enough for practical use.

It is also shown that the estimated parameters behave in a regular manner. For example the estimates of c and k are more or less linearly related and both c and k vary in an almost linear manner with time. This means that a projection model of fertility might be based on time specifications of c and k. This possibility has not been explored in the present paper. With respect to parameter d it is shown that it should not be interpreted as the starting age of fertility since estimates of d often are much below 15. It would appear that d essentially is related to the skewness of the observed fertility curve.

6.3 par1 and par2 as estimators of μ

Although not of intrinsic interest in a study of Swedish regional fertility, we have included a time series of Australian fertility schedules from between 1929 and 1970. This time series brings out some interesting features. For example, it shows that the ratios of mean parities par1 and par2 do not correlate very well with the mean of the schedules. More specifically, reasonably accurate estimation of μ from either par1 or par2 or, perhaps, from both par1 and par2 is not supported by this time series. A similar finding is obtained by studying the Swedish regional schedules from 1985-87. To our knowledge, it has not previously been shown that empirical fertility data only lend limited, or even doubtful, support for using par1 and/or par2 as estimators of μ . This is an important finding in the context of Brass estimation of child mortality in developing countries.

Another interesting feature brought out by the time series of Australian schedules is, as indeed one might expect, that the correlations between the fertility measures are much higher in a temporal than in a regional perspective. The significance of this finding is that one should distinguish between, on the one hand, regional and, on the other, temporal variation in the age-pattern of fertility. Putting it differently, the transition of one national age-pattern into another over time is a smoother or more gliding process than that of transitioning across regional age-patterns of fertility. This, of course, is especially true in the case of small regions.

6.4 Population projections at the municipal level

One of the reasons for undertaking the present study was to see if the municipal fertility schedules could or, in fact, should be used for making population projections at the municipal level. Because of the large variation in municipal fertility, it would appear that using the national fertility schedule invariably would result in an unnecessary error. After all, the individual municipal schedules have age-patterns much different from the national one. The extent to which use of the national schedule in municipal projections would distort the results has not been investigated. Indeed, this would be a formidable task.

But, if the purpose of the projections were to project (at the municipal level) the future number of children in need of day care centers and schooling, then it seems obvious that use should be made of the municipal schedules. In that case, however, it is essential that the schedules are graduated as otherwise the projected figures very well might deviate drastically from the observed future ones.

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APPENDIX

Variations in the age-pattern of fertility in Sweden around 1986

Area	R	Mean	Variance	Skewness	par1	par2
			_			
Sweden	1.79	28.92	24.94	0.6067	0.0427	0.2527
Stockholms län	1.66	29.55	27.45	0.2959	0.0527	0.2779
Upplands-Väsby	1.82	28.69	23.88	0.4339	0.0534	0.2662
Vallentuna	1.98	29.25	23.37	0.7378	0.0168	0.2164
Österåker	2.03	28.74	23.42	1.1193	0.0346	0.2178
Värmdö	2.00	28.71	27.43	0.9235	0.0540	0.2757
Järfälla	1.69	29.11	23.49	0.5663	0.0298	0.2322
Ekerö	2.11	29.16	19.56	1.0597	0.0130	0.1523
Huddinge	1.98	28.72	26.68	0.6603	0.0511	0.2850
Botkyrka	1.93	28.10	26.01	0.7048	0.0526	0.3150
Salem	1.85	29.12	20.86	0.6209	0.0164	0.2031
Haninge	1.85	28.48	26.27	0.7677	0.0436	0.2858
Tyresö	1.90	28.87	27.22	0.7497	0.0585	0.2641
Upplands-Bro	2.02	28.06	25.71	0.6698	0.0709	0.3059
Täby	1.85	30.18	21.39	0.2757	0.0414	0.1826
Danderyd	1.98	31.63	21.32	0.1282	0.0707	0.1565
Sollentuna	1.90	29.83	24.25	0.5396	0.0418	0.2046
Stockholm	1.48	30.35	29.17	0.0515	0.0650	0.2877
Södertälje	1.88	28.25	25.62	0.7240	0.0430	0.2991
Nacka	1.82	29.56	24.93	0.3114	0.0461	0.2574
Sundbyberg	1.57	29.47	24.22	0.2124	0.0424	0.2530
Solna	1.50	29.80	29.27	0.0306	0.0945	0.3152
Lidingö	1.63	30.99	22.45	0.0452	0.0539	0.1936
Vaxholm	1.89	29.49	25.46	0.7057	0.0038	0.2602
Norrtälje	1.97	28.57	25.18	0.7685	0.0435	0.2684
Sigtuna	1.77	28.55	24.70	0.6771	0.0455	0.2606
Nynäshamn	1.73	28.56	22.93	0.9153	0.0225	0.2281
Uppsala län	1.83	29.22	25.65	0.5077	0.0456	0.2538
Håbe	2.15	27.90	21.66	1.3388	0.0229	0.2428
Älvkarleby	1.67	27.85	22.02	1.1340	0.0059	0.2322
Tierp	1.99	27.90	21.30	1.0122	0.0290	0.2396
Uppsala	1.78	29.74	26.18	0.3003	0.0517	0.2549
Enköping	1.96	28.56	23.91	0.8827	0.0505	0.2409
Östhammar	2.10	28.23	24.51	1.1294	0.0438	0.2521

Table A1.-Regional fertility measures for Sweden, 1985-87

						_
Area	R 	Mean	Variance	Skewness	par1	par2
.					0.0400	
Södermanlands län	1.84	28.25	23.41	0.7530	0.0402	0.2647
Vingåker	1.98	27.68	23.46	0.8869	0.0420	0.2934
Nyköping	1.86	28.49	21.59	0.8538	0.0352	0.2190
Oxelösund	1.81	27.88	23.36	0.6953	0.0431	0.2940
Flen	1.85	28.19	23.07	0.9231	0.0169	0.2737
Katrineholm	1.87	28.21	22.83	0.6575	0.0474	0.2591
Eskilstuna	1.80	28.19	24.31	0.6968	0.0470	0.2819
Strängnäs	1.85	28.36	25.21	0.7714	0.0318	0.2827
Östergötlands län	1.86	28.65	24.29	0.7268	0.0447	0.2521
Ödeshög	2.61	29.12	29.17	1.7773	0.0133	0.2614
Ydre	2.42	28.41	28.65	2.3398	0.0254	0.2248
Kinda	2.21	28.25	25.07	1.0545	0.0359	0.2811
Boxholm	1.93	28.07	24.94	1.5117	0.0129	0.2247
Åtvidaberg	1.87	28.31	20.46	0.7250	0.0190	0.2241
Finspång	2.04	28.12	23.87	0.8418	0.0550	0.2642
Valdemarsvik	2.28	28.02	24.12	1.7021	0.0347	0.2328
Linköping	1.81	29.44	24.32	0.4304	0.0533	0.2336
Norrköping	1.75	28.29	23.87	0.6971	0.0434	0.2704
Söderköping	1.99	28.51	22.86	1.4262	0.0222	0.1897
Motala	2.00	28.01	23.38	0.8690	0.0507	0.2707
Vadstena	1.85	28.70	23.60	0.5990	0.0642	0.2445
Mjölby	1.94	28.28	22.89	1.1907	0.0258	0.2293
Jönköpings län	1.98	28.58	22.68	0.8279	0.0414	0.2309
Aneby	2.35	28.61	20.62	0.6881	0.0162	0.2374
Gnosjö	2.19	28.17	23.93	1.0648	0.0407	0.2732
Gislaved	2.10	28.07	24.27	0.8721	0.0556	0.2799
Vaggeryd	2.24	28.46	23.40	0.9769	0.0432	0.2376
Jönköping	1.91	28.83	22.52	0.6873	0.0368	0.2224
Nässjö	2.01	28.41	22.21	0.9676	0.0410	0.2272
Värnamo	1.88	28.72	23.58	0.7406	0.0650	0.2281
Sävsjö	2.28	28.39	23.58	1.2463	0.0372	0.2374
Vetlanda	1.88	28.54	22.00	1.0172	0.0340	0.2119
Eksjö	2.00	28.56	20.49	0.9947	0.0161	0.2015
Tranås	1.86	28.43	20.93	0.9669	0.0379	0.2074
Kronobergs län	1.91	28.83	22.73	0.7270	0.0457	0.2227
Uppvidinge	2.18	27.87	21.68	1.0220	0.0511	0.2503
Lessebo	2.08	28.10	22.73	0.7980	0.0510	0.2623
Tingsryd	2.26	28.49	24.48	0.9099	0.0278	0.2573
Alvesta	2.05	28.76	21.69	1.1763	0.0301	0.1946
Älmhult	1.93	28.79	22.72	0.9270	0.0388	0.2038
Markaryd	1.89	28.47	24.50	0.6122	0.0568	0.2701
Växjö	1.76	29.33	22.36	0.5210	0.0445	0.2092
Ljungby	1.98	28.50	22.01	0.9392	0.0548	0.2186

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Högsby2.1528.1521.610.70670.04810.2546Torsås2.2427.9522.301.10270.04200.2523Mörbylånga2.2528.5423.471.56020.04570.1981Hultsfred1.9728.3622.291.20890.05200.2215Mönsterås1.8427.8024.121.33320.01780.2687Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	'ea	R Me	an Variance	Skewness	par1	par2
Torsås2.2427.9522.301.10270.04200.2523Mörbylånga2.2528.5423.471.56020.04570.1981Hultsfred1.9728.3622.291.20890.05200.2215Mönsterås1.8427.8024.121.33320.01780.2687Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	ılmar län	1.90 28.4	7 23.17	0.9216	0.0345	0.2361
Mörbylånga2.2528.5423.471.56020.04570.1981Hultsfred1.9728.3622.291.20890.05200.2215Mönsterås1.8427.8024.121.33320.01780.2687Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	ögsby	2.15 28.1	5 21.61	0.7067	0.0481	0.2546
Hultsfred1.9728.3622.291.20890.05200.2215Mönsterås1.8427.8024.121.33320.01780.2687Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502		2.24 27.9	5 22.30	1.1027	0.0420	0.2523
Mönsterås1.8427.8024.121.33320.01780.2687Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502		2.25 28.5	4 23.47	1.5602	0.0457	0.1981
Emmaboda1.8828.4520.060.68670.05400.2028Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502				1.2089	0.0520	0.2215
Kalmar1.8329.0624.330.69920.03350.2260Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	önsterås	1.84 27.8	0 24.12	1.3332	0.0178	0.2687
Nybro1.8628.2121.560.91740.02560.2216Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	nmaboda	1.88 28.4	5 20.06	0.6867	0.0540	0.2028
Oskarshamn2.2528.5423.471.56020.04570.1981Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502		1.83 29.0	6 24.33	0.6992	0.0335	0.2260
Västervik1.8328.4022.870.76870.03010.2424Vimmerby1.9528.1623.020.91580.05540.2502	/bro	1.86 28.2	1 21.56	0.9174	0.0256	0.2216
Vimmerby 1.95 28.16 23.02 0.9158 0.0554 0.2502						0.1981
					0.0301	0.2424
Borgholm 2.14 29.20 25.52 1.2546 0.0122 0.1941						0.2502
	rgholm	2.14 29.2	0 25.52	1.2546	0.0122	0.1941
Gotlands län 1.84 28.72 24.02 0.6517 0.0414 0.2398	otlands län	1.84 28.7	2 24.02	0.6517	0.0414	0.2398
Gotland 1.84 28.72 24.02 0.6517 0.0414 0.2398	otland	1.84 28.7	2 24.02	0.6517	0.0414	0.2398
Blekinge län 1.89 28.26 23.04 0.8502 0.0316 0.2546	ekinge län	1.89 28.2	6 23.04	0.8502	0.0316	0.2546
Olofström 1.78 27.88 22.06 0.7824 0.0348 0.2531	ofström	1.78 27.8	8 22.06	0.7824	0.0348	0.2531
Karlskrona 1.96 28.48 24.01 0.8549 0.0259 0.2589	arlskrona	1.96 28.4	8 24.01	0.8549	0.0259	0.2589
Ronneby 1.94 27.76 22.02 0.9824 0.0367 0.2619	nneby	1.94 27.7	6 22.02	0.9824	0.0367	0.2619
Karlshamn 1.78 28.51 22.73 0.7948 0.0307 0.2412	rlshamn	1.78 28.5	1 22.73	0.7948	0.0307	0.2412
Sölvesborg 1.83 28.09 21.64 0.8042 0.0387 0.2468	lvesborg	1.83 28.0	9 21.64	0.8042	0.0387	0.2468
Kristianstads län 1.89 28.54 23.41 0.8552 0.0393 0.2389	istianstads län	1.89 28.5	4 23.41	0.8552	0.0393	0.2389
Östra Göinge 2.07 27.97 21.97 0.7517 0.0492 0.2717	stra Göinge	2.07 27.9	7 21.97	0.7517	0.0492	0.2717
		2.09 28.3	9 25.57	0.9294	0.0327	0.2669
		1.96 27.9	5 23.29	1.3723	0.0263	0.2407
			5 28.07	0.9137	0.0413	0.3019
Osby 2.10 28.02 22.56 1.3089 0.0284 0.2407	by	2.10 28.0	2 22.56	1.3089	0.0284	0.2407
Perstorp 1.94 27.79 22.68 0.7912 0.0428 0.2815		1.94 27.7	9 22.68	0.7912	0.0428	0.2815
Klippan 1.78 28.32 22.21 1.1952 0.0273 0.2203					0.0273	0.2203
Åstorp 1.88 27.52 22.63 1.1867 0.0415 0.2776				1.1867	0.0415	0.2776
Båstad 1.82 29.22 21.07 0.9492 0.0344 0.1635			2 21.07	0.9492	0.0344	0.1635
Kristianstad 1.86 28.80 22.86 0.6175 0.0485 0.2338	ristianstad		0 22.86		0.0485	0.2338
Simrishamn 1.71 28.85 23.18 0.7124 0.0148 0.2252	nrishamn			0.7124	0.0148	0.2252
Ängelholm 1.79 29.29 22.82 0.6591 0.0352 0.1960				0.6591	0.0352	
Hässleholm 1.97 28.45 23.44 1.0669 0.0410 0.2318	Ģ	1.97 28.4	5 23.44	1.0669	0.0410	0.2318

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Area	R	Mean	Variance	Skewne	ss par1	par2
Malmöhus län	1.73	29.02	24.87	0.5350	0.0455	0.2526
Svalöv	1.99	28.27	23.20	1.0645	0.0541	0.2318
Staffanstorp	1.92	28.71	19.08	0.9751	0.0253	0.1783
Burlöv	1.88	28.00	23.91	0.7579	0.0333	0.2850
Vellinge	2.07	29.07	19.38	1.1319	0.0077	0.1531
Bjuv	1.77	27.87	22.97	0.9794	0.0423	0.2579
Kävlinge	2.08	28.38	22.60	1.0880	0.0276	0.2276
Lomma	1.98	29.54	19.29	1.3327	0.0150	0.1018
Swedala	1.96	28.86	20.87	1.1467	0.0091	0.1746
Skurup	1.74	28.09	20.77	0.9178	0.0326	0.2238
Sjöbo	2.19	28.04	22.78	1.2303	0.0319	0.2532
Hörby	1.89	29.12	23.04	0.8681	0.0085	0.1963
Höör	1.99	28.93	23.45	0.9684	0.0287	0.2223
Malmö	1.60	29.05	27.95	0.3489	0.0647	0.2935
Lund	1.70	30.66	23.43	0.1580	0.0453	0.1997
Landskrona	1.76	28.16	26.03	0.7416	0.0812	0.3038
Helsingborg	1.68	28.79	24.53	0.4587	0.0449	0.2740
Höganäs	1.86	28.53	20.09	0.7464	0.0386	0.2150
Eslöv	2.00	28.37	22.85	1.1440	0.0280	0.2453
Ystad	1.75	28.82	21.18	1.0455	0.0145	0.1823
Trelleborg	1.84	28.24	23.46	1.0367	0.0296	0.2520
Hallands län	1.87	29.02	22.19	0.7702	0.0347	0.2071
Hylte	2.30	28.49	24.10	1.2943	0.0404	0.2267
Halmstad	1.73	29.04	24.04	0.6242	0.0500	0.2295
Laholm	2.13	28.42	21.32	1.2517	0.0276	0.2147
Falkenberg	1.90	28.99	21.99	0.9227	0.0474	0.1947
Varberg	1.87	28.92	21.73	0.6515	0.0272	0.2179
Kungsbacka	1.92	29.41	19.87	0.8188	0.0086	0.1643
Göteborgs och Bohus län	1.71	29.31	25.69	0.3730	0.0457	0.2605
Härryda	2.09	29.11	21.25	0.7817	0.0268	0.1979
Partille	1.92	29.21	23.16	0.6285	0.0292	0.2243
Öckerö	2.15	28.07	20.13	0.9443	0.0157	0.2340
Stenungsund	1.93	28.68	21.08	0.8185	0.0161	0.2123
Tjörn	2.33	28.57	22.03	1.0378	0.0493	0.2078
Orust	2.15	28.66	19.59	0.9053	0.0309	0.1852
Sotenäs	2.00	28.25	24.99	0.7246	0.0353	0.2721
Munkedal	2.14	28.57		1.1362	0.0276	0.2233
Tanum	1.91	28.95	25.39	1.2744	0.0502	0.1923
Göteborg	1.62	29.54	27.45	0.2200	0.0534	0.2857
Mölndal	1.80	29.40	24.19	0.3195	0.0442	0.2414
Kungälv	1.91	29.05	21.57	0.5658	0.0372	0.2107
Lysekil	1.79	28.35	25.28	1.0392	0.0441	0.2530
Uddevalla	1.84	28.77	23.23	0.6533	0.0403	0.2326
Strömstad	1.78	28.41	25.64	0.8723	0.0466	0.2736

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Area	R	Mean	Variance	Skewness	par1	par2
Ävlsborgs län	1.90	28.68	23.41	0.7440	0.0398	0.2393
Dals-Ed	1.81	28.76	28.84	0.9473	0.0745	0.2435
Färgelanda	2.11	27.96	21.93	0.9304	0.0489	0.2680
Ale	1.77	28.50	21.77	1.0898	0.0336	0.1950
Lerum	2.20	29.00	21.16	1.0258	0.0136	0.1896
Vårgårda	2.20	28.21	23.73	0.8134	0.0509	0.2669
Tranemo	2.03	27.84	19.72	0.9241	0.0307	0.2396
Bengtsfors	1.94	28.29	29.51	0.9362	0.0504	0.3045
Mellerud	1.96	28.59	25.77	1.0909	0.0274	0.2297
Lilla Edet	2.04	28.08	23.92	1.1857	0.0332	0.2544
Mark	1.97	28.71	23.42	0.9133	0.0328	0.2275
Svenljunga	1.99	28.48	21.98	0.9531	0.0294	0.2201
Herrljunga	2.20	28.19	21.03	0.8333	0.0203	0.2312
Vänersborg	1.92	28.67	23.17	0.6087	0.0375	0.2625
Trollhättan	1.79	28.32	22.75	0.7162	0.0338	0.2581
Alingsås	2.05	29.14	24.80	0.6881	0.0403	0.2351
Borås	1.74	28.84	23.59	0.4463	0.0624	0.2465
Ulricehamn	1.91	29.34	24.39	0.7234	0.0401	0.2138
Åmål	1.71	28.55	21.41	0.8042	0.0264	0.2205
Skaraborgs län	1.95	28.45	23.27	0.8772	0.0391	0.2443
Grästorp	2.23	28.48	26.71	0.8657	0.0878	0.2703
Essunga	2.02	28.78	22.69	0.6162	0.0137	0.2592
Mullsjö	2.33	28.17	23.17	1.0843	0.0474	0.2521
Habo	2.34	28.10	20.69	1.3523	0.0187	0.2188
Karlsborg	2.18	27.67	20.20	0.9264	0.0658	0.2422
Gullspång	2.26	27.51	21.30	0.8457	0.1010	0.2805
Vars	2.10	28.62	24.27	1.1213	0.0664	0.2367
Götene	1.86	28.39	23.18	0.9706	0.0247	0.2389
Tibro	1.99	28.57	20.80	0.8438	0.0203	0.2019
Töreboda	1.95	27.74	24.71	1.1986	0.0404	0.2783
Mariestad	1.81	28.21	23.62	0.9757	0.0340	0.2662
Lidköping	1.79	28.52	23.23	0.6661	0.0326	0.2458
Skara	1.89	28.97	24.99	0.6586	0.0448	0.2571
Skövde	1.87	28.60	22.74	0.7529	0.0375	0.2360
Hjo	1.96	28.40	25.01	0.7389	0.0326	0.2726
Tidaholm	2.01	28.67	25.95	1.0699	0.0247	0.2446
Falköping	1.98	28.46	21.71	1.0864	0.0146	0.2199

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Area	R	Mean	Variance	Skewness	par1	par2
Värmlands län	1.81	28.53	23.95	0.7283	0.0410	0.2552
Kil	2.02	27.82	19.53	1.0378	0.0194	0.2452
Eda	1.96	28.07	25.20	1.2024	0.0349	0.2614
Torsby	1.88	28.40	26.34	0.7751	0.0464	0.2823
Storfors	2.05	27.57	25.81	1.1826	0.0481	0.3019
Hammarö	2.01	28.43	21.58	1.0378	0.0378	0.2012
Munkfors	2.11	27.79	20.12	0.7554	0.0146	0.2722
Forshaga	1.98	27.98	22.56	1.1875	0.0268	0.2483
Grums	1.74	27.54	22.35	0.6617	0.0503	0.2912
Årjäng	1.82	28.59	26.24	0.6986	0.0929	0.2601
Sunne	1.86	28.87	24.93	0.7791	0.0234	0.2482
Karlstad	1.69	29.11	23.46	0.4646	0.0458	0.2390
Kristinehamn	1.80	28.12	23.09	0.9552	0.0351	0.2628
Filipstad	1.67	28.17	27.40	0.7725	0.0419	0.3020
Hagfors	1.83	28.41	24.54	1.0108	0.0390	0.2486
Arvika	1.88	28.70	25.51	0.6823	0.0527	0.2708
Säffle	1.91	28.52	22.44	0.7428	0.0197	0.2393
Örebro län	1.81	28.43	24.38	0.7522	0.0403	0.2708
Laxå	1.83	27.61	25.63	1.2936	0.0473	0.2984
Hallsberg	1.99	28.32	23.43	0.9525	0.0268	0.2607
Degerfors	1.88	27.54	22.79	1.0900	0.0382	0.2960
Hällefors	1.95	27.91	25.73	0.7319	0.0473	0.3377
Ljusnarsberg	1.86	27.58	26.11	0.5897	0.0648	0.3619
Örebro	1.75	28.81	24.10	0.5629	0.0420	0.2602
Kumla	1.89	28.09	23.66	1.2564	0.0219	0.2487
Askersund	1.88	28.28	26.26	1.3985	0.0245	0.2579
Karlskoga	1.77	28.40	21.92	0.5195	0.0449	0.2547
Nora	1.77	28.86	26.41	0.7302	0.0359	0.2615
Lindesberg	1.93	27.84	25.48	1.2413	0.0405	0.2840
Västmanlands län	1.81	28.39	23.62	0.8428	0.0330	0.2548
Skinnskatteberg	2.18	27.78	22.89	1.2952	0.0686	0.2398
Surahammar	1.93	28.33	22.74	0.8180	0.0395	0.2479
Heby	2.34	28.00	24.01	1.4435	0.0247	0.2586
Kungsör	1.73	27.50	20.12	1.0738	0.0218	0.2693
Hallstahammar	1.69	28.20	26.01	1.0005	0.0369	0.2634
Norberg	1.93	27.47	20.79	1.0195	0.0429	0.2760
Västerås	1.73	28.77	23.57	0.6699	0.0333	0.2478
Sala	1.95	28.16	23.04	0.9925	0.0430	0.2445
Fagersta	1.76	28.18	25.37	0.9553	0.0162	0.2695
Köping	1.81	28.04	23.15	0.8771	0.0247	0.2769
Arboga	1.90	28.06	23.09	1.1721	0.0306	0.2441

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Area	R	Mean	Variance	Skewness	par1	par2
Kopparbergs län	1.95	28.69	24.40	0.8265	0.0405	0.2435
Vansbro	1.89	28.73	26.02	1.3494	0.0104	0.2123
Malung	1.71	28.33	21.20	0.9185	0.0351	0.2252
Gagner	1.86	28.77	24.21	0.9333	0.0370	0.2226
Leksand	2.09	29.95	22.11	0.5114	0.0205	0.1679
Rättvik	1.83	28.66	22.29	0.6478	0.0650	0.2292
Orsa	1.99	29.73	33.49	0.7967	0.0695	0.2805
Älvdalen	2.25	28.13	26.09	1.5522	0.0274	0.2740
Smedjebacken	2.08	27.67	22.58	0.6657	0.0407	0.3089
Mora	1.88	29.29	25.34	0.6386	0.0394	0.2214
Falun	1.93	29.11	24.85	0.6514	0.0392	0.2363
Borlänge	1.90	28.63	22.94	0.7983	0.0434	0.2301
Säter	2.30	28.25	25.61	1.0076	0.0654	0.2612
Hedemora	2.10	28.10	23.43	0.9544	0.0404	0.2739
Avesta	1.94	28.17	23.60	1.0951	0.0411	0.2536
Ludvika	1.84	28.40	23.68	0.8509	0.0294	0.2545
Gävleborgs län	1.78	28.48	24.13	0.7968	0.0347	0.2542
Ockelbo	1.97	28.56	24.83	1.0764	0.0424	0.2428
Hofors	1.93	28.09	24.45	1.1861	0.0073	0.2711
Ovanåker	1.98	28.16	24.95	1.2545	0.0318	0.2596
Nordanstig	2.25	28.22	23.56	0.8728	0.0462	0.2554
Ljusdal	1.91	28.30	23.96	1.0257	0.0164	0.2548
Gävle	1.69	28.71	23.63	0.6261	0.0355	0.2434
Sandviken	1.75	27.91	21.02	0.7166	0.0380	0.2595
Söderhamn	1.73	28.13	25.96	1.0283	0.0354	0.2862
Bollnäs	1.78	28.75	24.98	0.9147	0.0250	0.2401
Hudiksvall	1.81	28.92	25.33	0.6167	0.0488	0.2483
Västnorrlands län	1.82	28.57	22.90	0.7539	0.0408	0.2352
Ånge	1.88	28.23	27.33	1.0762	0.0328	0.2940
Timrå	1.72	28.05	22.62	0.5796	0.0581	0.2666
Härnösand	1.82	28.49	24.06	0.8205	0.0401	0.2504
Sundsvall	1.70	28.66	23.59	0.6389	0.0456	0.2458
Kramfors	1.77	28.59	22.76	0.6630	0.0396	0.2352
Sollefteå	2.09	28.50	22.65	0.7270	0.0372	0.2493
Örnsköldsvik	1.97	28.68	20.65	1.0370	0.0298	0.1832
Jämtlands län	1.91	29.03	25.40	0.5417	0.0440	0.2599
Ragunda	2.18	28.83	24.57	1.0046	0.0137	0.2313
Bräcke	2.10	28.47	28.33	0.8611	0.0474	0.3001
Krokom	2.39	28.97	24.98	0.9891	0.0439	0.2353
Strömsund	2.21	28.49	26.00	1.0907	0.0639	0.2517
Åre	2.10	28.76	25.08	0.6108	0.0345	0.2795
Berg	2.39	28.69	23.27	0.9128	0.0078	0.2469
Härjedalen	1.86	28.88	26.26	0.7098	0.0266	0.2553
Östersund	1.67	29.34	24.78	0.2399	0.0493	0.2631

Table A1.-Regional fertility measures for Sweden, 1985-87, Cont'd

Area	R	Mean	Variance	Skewness	par1	par2
Västerbottens län	1.92	29.19	23.65	0.8662	0.0304	0.2033
Nordmaling	2.34	28.78	25.79	0.8758	0.0383	0.2538
Bjurholm	2.52	29.00	23.99	1.5124	0.0601	0.1746
Vindeln	2.38	29.32	22.10	0.9686	0.0196	0.1788
Robertsfors	2.40	28.58	25.42	1.7628	0.0352	0.2140
Norsjö	2.49	28.04	24.51	1.9801	0.0082	0.2418
Malấ	1.91	29.45	24.90	0.8681	0.0775	0.1732
Storuman	2.02	29.12	25.17	1.2190	0.0111	0.1971
Sorsele	2.41	29.00	32.79	1.6191	0.0141	0.2964
Dorotea	2.05	27.91	21.86	0.9675	0.0054	0.2730
Vännäs	2.20	28.34	20.78	1.2038	0.0338	0.1827
Vilhelmina	2.33	29.05	28.50	1.1603	0.0416	0.2516
Åsele	1.93	28.89	22.78	1.0314	0.0108	0.1786
Umeå	1.81	29.63	23.87	0.6352	0.0326	0.2031
Lycksele	2.00	28.71	23.96	0.9168	0.0565	0.2235
Skellefteå	1.83	28.96	21.18	0.9698	0.0219	0.1786
Norrbottens län	1.83	28.74	24.81	0.8512	0.0368	0.2440
Arvidsjaur	1.85	28.20	20.14	0.9446	0.0398	0.2060
Arjeplog	2.61	28.03	24.35	0.6734	0.0717	0.3103
Jokkmokk	1.83	28.68	29.24	0.6859	0.0614	0.3100
Överkalix	1.81	27.53	19.19	0.4229	0.0426	0.2700
Kalix	2.00	28.13	28.83	1.3094	0.0482	0.3119
Övertorneå	2.20	28.24	26.34	1.1497	0.0460	0.2595
Pajala	2.28	29.32	32.92	1.3842	0.0747	0.2351
Gällivare	1.76	28.86	24.47	0.7882	0.0285	0.2338
Älvsbyn	2.20	28.39	20.02	0.7924	0.0167	0.2097
Luleå	1.68	29.04	24.57	0.5738	0.0333	0.2380
Piteå	1.85	28.71	20.61	1.0748	0.0204	0.1917
Boden	1.78	28.75	24.70	0.8183	0.0297	0.2440
Haparanda	1.93	28.79	27.90	1.2160	0.0214	0.2737
Kiruna	1.83	28.91	27.25	0.8224	0.0387	0.2615

Municipality	R	Mean	Variance	Skewness	par1	par2
Ödeshög	2.61	29.12	29.17	1.7773	0.0133	0.2614
Arjeplog	2.61	28.03	24.35	0.6734	0.0717	0.3103
Bjurholm	2.52	29.00	23.99	1.5124	0.0601	0.1746
Norsjö	2.49	28.04	24.51	1.9801	0.0082	0.2418
Ydre	2.42	28.41	28.65	2.3398	0.0254	0.2248
Sorsele	2.41	29.00	32.79	1.6191	0.0141	0.2964
Robertsfors	2.40	28.58	25.42	1.7628	0.0352	0.2140
Krokom	2.39	28.97	24.98	0.9891	0.0439	0.2353
Berg	2.39	28.69	23.27	0.9128	0.0078	0.2469
Vindeln	2.38	29.32	22.10	0.9686	0.0196	0.1788
Aneby	2.35	28.61	20.62	0.6881	0.0162	0.2374
Nordmaling	2.34	28.78	25.79	0.8758	0.0383	0.2538
Habo	2.34	28.10	20.69	1.3523	0.0187	0.2188
Heby	2.34	28.00	24.01	1.4435	0.0247	0.2586
Vilhelmina	2.33	29.05	28.50	1.1603	0.0416	0.2516
Tjörn	2.33	28.57	22.03	1.0378	0.0493	0.2078
Mullsjö	2.33	28.17	23.17	1.0843	0.0474	0.2521
Hylte	2.30	28.49	24.10	1.2943	0.0404	0.2267
Säter	2.30	28.25	25.61	1.0076	0.0654	0.2612
Pajala	2.28	29.32	32.92	1.3842	0.0747	0.2351
Sävsjö	2.28	28.39	23.58	1.2463	0.0372	0.2374
Valdemarsvik	2.28	28.02	24.12	1.7021	0.0347	0.2328
Tingsryd	2.26	28.49	24.48	0.9099	0.0278	0.2573
Gullspång	2.26	27.51	21.30	0.8457	0.1010	0.2805
Mörbylånga	2.25	28.54	23.47	1.5602	0.0457	0.1981
Oskarshamn	2.25	28.54	23.47	1.5602	0.0457	0.1981
Nordanstig	2.25	28.22	23.56	0.8728	0.0462	0.2554
Älvdalen	2.25	28.13	26.09	1.5522	0.0274	0.2740
Vaggeryd	2.24	28.46	23.40	0.9769	0.0432	0.2376
Torsås	2.24	27.95	22.30	1.1027	0.0420	0.2523
Grästorp	2.23	28.48	26.71	0.8657	0.0878	0.2703
Strömsund	2.21	28.49	26.00	1.0907	0.0639	0.2517
Kinda	2.21	28.25	25.07	1.0545	0.0359	0.2811
Lerum	2.20	29.00	21.16	1.0258	0.0136	0.1896
Älvsbyn	2.20	28.39	20.02	0.7924	0.0167	0.2097
Vännäs	2.20	28.34	20.78	1.2038	0.0338	0.1827
Övertorneå	2.20	28.24	26.34	1.1497	0.0460	0.2595
Vårgårda	2.20	28.21	23.73	0.8134	0.0509	0.2669
Herrljunga	2.20	28.19	21.03	0.8333	0.0203	0.2312
Gnosjö	2.19	28.17	23.93	1.0648	0.0407	0.2732
Sjöbo	2.19	28.04	22.78	1.2303	0.0319	0.2532
Ragunda	2.18	28.83	24.57	1.0046	0.0137	0.2313
Uppvidinge	2.18	27.87	21.68	1.0220	0.0511	0.2503
Skinnskatteberg	2.18	27.78	22.89	1.2952	0.0686	0.2398
Karlsborg	2.18	27.67	20.20	0.9264	0.0658	0.2422
Orust	2.15	28.66	19.59	0.9053	0.0309	0.1852
Högsby	2.15	28.15	21.61	0.7067	0.0481	0.2546
Öckerö	2.15	28.07	20.13	0.9443	0.0157	0.2340
Håbe	2.15	27.90	20.13	1.3388	0.0229	0.2428
Borgholm	2.13 2.14	27.90	25.52	1.3388	0.0229	0.2428
DOIGHOIM	2.14	27.20	20.02	1.2070	0.0144	0.1741

	R	Mean	Variance	Skewness	par1	par2
Munkedal	2.14	28.57	22.81	1.1362	0.0276	0 2222
Laholm	2.14	28.37	22.81	1.1362	0.0276 0.0276	0.2233 0.2147
Ekerö	2.13	29.16	19.56	1.0597	0.0278	0.2147
Färgelanda	2.11	29.16	21.93	0.9304	0.0130	0.1525
Munkfors	2.11	27.90	20.12	0.9304	0.0489	0.2660
Åre	2.11	28.76	25.08	0.7354	0.0140	0.2722
Vars	2.10	28.62	23.08	1.1213	0.0343	0.2793
Bräcke	2.10	28.62	28.33	0.8611	0.0004	0.2307
Östhammar	2.10	28.23	20.55 24.51	1.1294	0.0474	0.2521
Hedemora	2.10	28.10	23.43	0.9544	0.0404	0.2739
Gislaved	2.10	28.07	23.45	0.9344	0.0556	0.2739
Osby	2.10	28.02	22.56	1.3089	0.0350	0.2799
Leksand	2.09	29.95	22.30	0.5114	0.0205	0.2407
Härryda	2.09	29.11	21.25	0.7817	0.0265	0.1079
Sollefteå	2.09	28.50	22.65	0.7270	0.0200	0.2493
Örkelljunga	2.09	28.39	25.57	0.9294	0.0372	0.2493
Kävlinge	2.09	28.38	22.60	1.0880	0.0327	0.2009
Lessebo	2.08	28.10	22.73	0.7980	0.0270	0.2623
Smedjebacken	2.08	27.67	22.58	0.6657	0.0310	0.2023
Vellinge	2.07	29.07	19.38	1.1319	0.0407	0.3089
Östra Göinge	2.07	27.97	21.97	0.7517	0.0492	0.1331
Alingsås	2.05	29.14	24.80	0.6881	0.0492	0.2351
Alvesta	2.05	29.14	21.69	1.1763	0.0403	0.2351
Dorotea	2.05	27.91	21.86	0.9675	0.0054	0.1940
Storfors	2.05	27.57	25.81	1.1826	0.0034	0.2750
Finspång	2.03	28.12	23.87	0.8418	0.0550	0.2642
Lilla Edet	2.04	28.08	23.92	1.1857	0.0332	0.2544
Österåker	2.03	28.74	23.42	1.1193	0.0346	0.2178
Tranemo	2.03	27.84	19.72	0.9241	0.0307	0.2396
Storuman	2.02	29.12	25.17	1.2190	0.0307	0.1971
Essunga	2.02	28.78	22.69	0.6162	0.0137	0.2592
Upplands-Bro	2.02	28.06	25.71	0.6698	0.0709	0.3059
Kil	2.02	27.82	19.53	1.0378	0.0194	0.2452
Tidaholm	2.01	28.67	25.95	1.0699	0.0247	0.2446
Hammarö	2.01	28.43	21.58	1.0378	0.0378	0.2012
Nässjö	2.01	28.41	22.21	0.9676	0.0410	0.2272
Värmdö	2.00	28.71	27.43	0.9235	0.0540	0.2757
Lycksele	2.00	28.71	23.96	0.9168	0.0565	0.2235
Eksjö	2.00	28.56	20.49	0.9947	0.0161	0.2015
Éslöv	2.00	28.37	22.85	1.1440	0.0280	0.2453
Sotenäs	2.00	28.25	24.99	0.7246	0.0353	0.2721
Kalix	2.00	28.13	28.83	1.3094	0.0482	0.3119
Motala	2.00	28.01	23.38	0.8690	0.0507	0.2707
Orsa	1.99	29.73	33.49	0.7967	0.0695	0.2805
Höör	1.99	28.93	23.45	0.9684	0.0287	0.2223
Tibro	1.99	28.57	20.80	0.8438	0.0203	0.2019
Söderköping	1.99	28.51	22.86	1.4262	0.0222	0.1897
Svenljunga	1.99	28.48	21.98	0.9531	0.0294	0.2201
Hallsberg	1.99	28.32	23.43	0.9525	0.0268	0.2607
Svalöv	1.99	28.27	23.20	1.0645	0.0541	0.2318
Svalov	1.99	28.27	23.20	1.0645	0.0541	0.2318

Table A2.-Regional fertility ranked with respect to the total fertility rate and the mean age of the fertility schedule, Cont'd

Municipality	R	Mean	Variance	Skewness	par1	par2
Tierp	1.99	27.90	21.30	1.0122	0.0290	0.2396
Danderyd	1.98	31.63	21.32	0.1282	0.0707	0.1565
Lomma	1.98	29.54	19.29	1.3327	0.0150	0.1018
Vallentuna	1.98	29.25	23.37	0.7378	0.0168	0.2164
Huddinge	1.98	28.72	26.68	0.6603	0.0511	0.2850
Ljungby	1.98	28.50	22.01	0.9392	0.0548	0.2186
Falköping	1.98	28.46	21.71	1.0864	0.0146	0.2199
Ovanåker	1.98	28.16	24.95	1.2545	0.0318	0.2596
Forshaga	1.98	27.98	22.56	1.1875	0.0268	0.2483
Vingåker	1.98	27.68	23.46	0.8869	0.0420	0.2934
Mark	1.97	28.71	23.42	0.9133	0.0328	0.2275
Örnsköldsvik	1.97	28.68	20.65	1.0370	0.0298	0.1832
Norrtälje	1.97	28.57	25.18	0.7685	0.0435	0.2684
Ockelbo	1.97	28.56	24.83	1.0764	0.0424	0.2428
Hässleholm	1.97	28.45	23.44	1.0669	0.0410	0.2318
Hultsfred	1.97	28.36	22.29	1.2089	0.0520	0.2215
Swedala	1.96	28.86	20.87	1.1467	0.0091	0.1746
Mellerud	1.96	28.59	25.77	1.0909	0.0274	0.2297
Enköping	1.96	28.56	23.91	0.8827	0.0505	0.2409
Karlskrona	1.96	28.48	24.01	0.8549	0.0259	0.2589
Нјо	1.96	28.40	25.01	0.7389	0.0326	0.2726
Bromölla	1.96	28.25	28.07	0.9137	0.0413	0.3019
Eda	1.96	28.07	25.20	1.2024	0.0349	0.2614
Tomelilla	1.96	27.95	23.29	1.3723	0.0263	0.2407
Vimmerby	1.95	28.16	23.02	0.9158	0.0554	0.2502
Sala	1.95	28.16	23.04	0.9925	0.0430	0.2445
Hällefors	1.95	27.91	25.73	0.7319	0.0473	0.3377
Töreboda	1.95	27.74	24.71	1.1986	0.0404	0.2783
Bengtsfors	1.94	28.29	29.51	0.9362	0.0504	0.3045
Mjölby	1.94	28.28	22.89	1.1907	0.0258	0.2293
Avesta	1.94	28.17	23.60	1.0951	0.0411	0.2536
Perstorp	1.94	27.79	22.68	0.7912	0.0428	0.2815
Ronneby	1.94	27.76	22.02	0.9824	0.0367	0.2619
Falun	1.93	29.11	24.85	0.6514	0.0392	0.2363
Åsele	1.93	28.89	22.78	1.0314	0.0108	0.1786
Älmhult	1.93	28.79	22.72	0.9270	0.0388	0.2038
Haparanda	1.93	28.79	27.90	1.2160	0.0214	0.2737
Stenungsund	1.93	28.68	21.08	0.8185	0.0161	0.2123
Surahammar	1.93	28.33	22.74	0.8180	0.0395	0.2479
Botkyrka	1.93	28.10	26.01	0.7048	0.0526	0.3150
Hofors	1.93	28.09	24.45	1.1861	0.0073	0.2711
Boxholm	1.93	28.07	24.94	1.5117	0.0129	0.2247
Lindesberg	1.93	27.84	25.48	1.2413	0.0405	0.2840
Norberg	1.93	27.47	20.79	1.0195	0.0429	0.2760
Kungsbacka	1.92	29.41	19.87	0.8188	0.0086	0.1643
Partille	1.92	29.21	23.16	0.6285	0.0292	0.2243
Staffanstorp	1.92	28.71	19.08	0.9751	0.0253	0.1783
Vänersborg	1.92	28.67	23.17	0.6087	0.0375	0.2625
Malå	1.91	29.45	24.90	0.8681	0.0775	0.1732
Ulricehamn	1.91	29.34	24.39	0.7234	0.0401	0.2138

Table A2.-Regional fertility ranked with respect to the total fertility rate and the mean age of the fertility schedule, Cont'd

Municipality	R	Mean	Variance	Skewness	par1	par2
Kungalu	1.91	29.05	21.57	0.5658	0.0372	0.2107
Kungälv Tanum	1.91	29.05	25.39	1.2744	0.0572	0.1923
		28.93	23.39	0.6873	0.0368	0.1923
Jönköping Säffle	1.91	28.52	22.32	0.8873	0.0368	0.2224
	1.91	28.32	23.96	1.0257	0.0197	0.2548
Ljusdal Sallontuna	1.91	28.30	23.96	0.5396	0.0164	0.2348
Sollentuna	1.90	29.83	24.23 21.99	0.9227	0.0418	0.2046
Falkenberg	1.90		27.22			
Tyresö	1.90	28.87		0.7497	0.0585	0.2641
Borlänge	1.90	28.63	22.94	0.7983	0.0434	0.2301
Arboga	1.90	28.06	23.09	1.1721	0.0306	0.2441
Vaxholm	1.89	29.49	25.46	0.7057	0.0038	0.2602
Hörby	1.89	29.12	23.04	0.8681	0.0085	0.1963
Skara	1.89	28.97	24.99	0.6586	0.0448	0.2571
Vansbro	1.89	28.73	26.02	1.3494	0.0104	0.2123
Markaryd	1.89	28.47	24.50	0.6122	0.0568	0.2701
Kumla	1.89	28.09	23.66	1.2564	0.0219	0.2487
Mora	1.88	29.29	25.34	0.6386	0.0394	0.2214
Värnamo	1.88	28.72	23.58	0.7406	0.0650	0.2281
Arvika	1.88	28.70	25.51	0.6823	0.0527	0.2708
Vetlanda	1.88	28.54	22.00	1.0172	0.0340	0.2119
Emmaboda	1.88	28.45	20.06	0.6867	0.0540	0.2028
Torsby	1.88	28.40	26.34	0.7751	0.0464	0.2823
Askersund	1.88	28.28	26.26	1.3985	0.0245	0.2579
Södertälje	1.88	28.25	25.62	0.7240	0.0430	0.2991
Ånge	1.88	28.23	27.33	1.0762	0.0328	0.2940
Burlöv	1.88	28.00	23.91	0.7579	0.0333	0.2850
Degerfors	1.88	27.54	22.79	1.0900	0.0382	0.2960
Åstorp	1.88	27.52	22.63	1.1867	0.0415	0.2776
Varberg	1.87	28.92	21.73	0.6515	0.0272	0.2179
Skövde	1.87	28.60	22.74	0.7529	0.0375	0.2360
Åtvidaberg	1.87	28.31	20.46	0.7250	0.0190	0.2241
Katrineholm	1.87	28.21	22.83	0.6575	0.0474	0.2591
Härjedalen	1.86	28.88	26.26	0.7098	0.0266	0.2553
Sunne	1.86	28.87	24.93	0.7791	0.0234	0.2482
Kristianstad	1.86	28.80	22.86	0.6175	0.0485	0.2338
Gagnef	1.86	28.77	24.21	0.9333	0.0370	0.2226
Höganäs	1.86	28.53	20.09	0.7464	0.0386	0.2150
Nyköping	1.86	28.49	21.59	0.8538	0.0352	0.2190
Tranås	1.86	28.43	20.93	0.9669	0.0379	0.2074
Götene	1.86	28.39	23.18	0.9706	0.0247	0.2389
Nybro	1.86	28.21	21.56	0.9174	0.0256	0.2216
Ljusnarsberg	1.86	27.58	26.11	0.5897	0.0648	0.3619
Täby	1.85	30.18	21.39	0.2757	0.0414	0.1826
Salem	1.85	29.12	20.86	0.6209	0.0164	0.2031
Piteå	1.85	28.71	20.61	1.0748	0.0204	0.1917
Vadstena	1.85	28.70	23.60	0.5990	0.0642	0.2445
Haninge	1.85	28.48	26.27	0.7677	0.0436	0.2858
Strängnäs	1.85	28.36	25.21	0.7714	0.0318	0.2827
Arvidsjaur	1.85	28.20	20.14	0.9446	0.0398	0.2060
Flen	1.85	28.19	23.07	0.9440	0.0398	0.2737

Table A2.-Regional fertility ranked with respect to the total fertility rate and the mean age of the fertility schedule, Cont'd

Municipality	R	Mean	Variance	Skewness	par1	par2
Uddevalla	1.84	28.77	23.23	0.6533	0.0403	0.2326
Gotland	1.84	28.72	24.02	0.6517	0.0414	0.2398
Ludvika	1.84	28.40	23.68	0.8509	0.0294	0.2545
Trelleborg	1.84	28.24	23.46	1.0367	0.0296	0.2520
Mönsterås	1.84	27.80	24.12	1.3332	0.0178	0.2687
Kalmar	1.83	29.06	24.33	0.6992	0.0335	0.2260
Skellefteå	1.83	28.96	21.18	0.9698	0.0219	0.1786
Kiruna	1.83	28.91	27.25	0.8224	0.0387	0.2615
Jokkmokk	1.83	28.68	29.24	0.6859	0.0614	0.3100
Rättvik	1.83	28.66	22.29	0.6478	0.0650	0.2292
Hagfors	1.83	28.41	24.54	1.0108	0.0390	0.2486
Västervik	1.83	28.40	22.87	0.7687	0.0301	0.2424
Sölvesborg	1.83	28.09	21.64	0.8042	0.0387	0.2468
Laxå	1.83	27.61	25.63	1.2936	0.0473	0.2984
Nacka	1.82	29.56	24.93	0.3114	0.0461	0.2574
Båstad	1.82	29.22	21.07	0.9492	0.0344	0.1635
Upplands-Väsby	1.82	28.69	23.88	0.4339	0.0534	0.2662
Årjäng	1.82	28.59	26.24	0.6986	0.0929	0.2601
Härnösand	1.82	28.49	24.06	0.8205	0.0401	0.2504
Umeå	1.81	29.63	23.87	0.6352	0.0326	0.2031
Linköping	1.81	29.44	24.32	0.4304	0.0533	0.2336
Hudiksvall	1.81	28.92	25.33	0.6167	0.0488	0.2483
Dals-Ed	1.81	28.76	28.84	0.9473	0.0745	0.2435
Mariestad	1.81	28.21	23.62	0.9757	0.0340	0.2662
Köping	1.81	28.04	23.15	0.8771	0.0247	0.2769
Oxelösund	1.81	27.88	23.36	0.6953	0.0431	0.2940
Överkalix	1.81	27.53	19.19	0.4229	0.0426	0.2700
Mölndal	1.80	29.40	24.19	0.3195	0.0442	0.2414
Eskilstuna	1.80	28.19	24.31	0.6968	0.0470	0.2819
Kristinehamn	1.80	28.12	23.09	0.9552	0.0351	0.2628
Ängelholm	1.79	29.29	22.82	0.6591	0.0352	0.1960
Lidköping	1.79	28.52	23.23	0.6661	0.0326	0.2458
Lysekil	1.79	28.35	25.28	1.0392	0.0441	0.2530
Trollhättan	1.79	28.32	22.75	0.7162	0.0338	0.2581
Uppsala	1.78	29.74	26.18	0.3003	0.0517	0.2549
Bollnäs	1.78	28.75	24.98	0.9147	0.0250	0.2401
Boden	1.78	28.75	24.70	0.8183	0.0297	0.2440
Karlshamn	1.78	28.51	22.73	0.7948	0.0307	0.2412
Strömstad	1.78	28.41	25.64	0.8723	0.0466	0.2736
Klippan	1.78	28.32	22.21	1.1952	0.0273	0.2203
Olofström	1.78	27.88	22.06	0.7824	0.0348	0.2531
Nora	1.77	28.86	26.41	0.7302	0.0359	0.2615
Kramfors	1.77	28.59	22.76	0.6630	0.0396	0.2352
Sigtuna	1.77	28.55	24.70	0.6771	0.0455	0.2606
Ale	1.77	28.50	21.77	1.0898	0.0336	0.1950
Karlskoga	1.77	28.40	21.92	0.5195	0.0449	0.2547
Bjuv	1.77	27.87	22.97	0.9794	0.0423	0.2579
Växjö	1.76	29.33	22.36	0.5210	0.0445	0.2092
Gällivare	1.76	28.86	24.47	0.7882	0.0285	0.2338
Fagersta	1.76	28.18	25.37	0.9553	0.0162	0.2695
	1.70	20.10		0.7000		0.2070

Table A2Regional fertility ranked with respect to the total fertility rate and
the mean age of the fertility schedule, Cont'd

Municipality	R	Mean	Variance	Skewness	par1	par2

Landskrona	1.76	28.16	26.03	0.7416	0.0812	0.3038
Ystad	1.75	28.82	21.18	1.0455	0.0145	0.1823
Örebro	1.75	28.81	24.10	0.5629	0.0420	0.2602
Norrköping	1.75	28.29	23.87	0.6971	0.0434	0.2704
Sandviken	1.75	27.91	21.02	0.7166	0.0380	0.2595
Borås	1.74	28.84	23.59	0.4463	0.0624	0.2465
Skurup	1.74	28.09	20.77	0.9178	0.0326	0.2238
Grums	1.74	27.54	22.35	0.6617	0.0503	0.2912
Halmstad	1.73	29.04	24.04	0.6242	0.0500	0.2295
Västerås	1.73	28.77	23.57	0.6699	0.0333	0.2478
Nynäshamn	1.73	28.56	22.93	0.9153	0.0225	0.2281
Söderhamn	1.73	28.13	25.96	1.0283	0.0354	0.2862
Kungsör	1.73	27.50	20.12	1.0738	0.0218	0.2693
Timrå	1.72	28.05	22.62	0.5796	0.0581	0.2666
Simrishamn	1.71	28.85	23.18	0.7124	0.0148	0.2252
Åmål	1.71	28.55	21.41	0.8042	0.0264	0.2205
Malung	1.71	28.33	21.20	0.9185	0.0351	0.2252
Lund	1.70	30.66	23.43	0.1580	0.0453	0.1997
Sundsvall	1.70	28.66	23.59	0.6389	0.0456	0.2458
Järfälla	1.69	29.11	23.49	0.5663	0.0298	0.2322
Karlstad	1.69	29.11	23.46	0.4646	0.0458	0.2390
Gävle	1.69	28.71	23.63	0.6261	0.0355	0.2434
Hallstahammar	1.69	28.20	26.01	1.0005	0.0369	0.2634
Luleå	1.68	29.04	24.57	0.5738	0.0333	0.2380
Helsingborg	1.68	28.79	24.53	0.4587	0.0449	0.2740
Östersund	1.67	29.34	24.78	0.2399	0.0493	0.2631
Filipstad	1.67	28.17	27.40	0.7725	0.0419	0.3020
Älvkarleby	1.67	27.85	22.02	1.1340	0.0059	0.2322
Lidingö	1.63	30.99	22.45	0.0452	0.0539	0.1936
Göteborg	1.62	29.54	27.45	0.2200	0.0534	0.2857
Malmö	1.60	29.05	27.95	0.3489	0.0647	0.2935
Sundbyberg	1.57	29.47	24.22	0.2124	0.0424	0.2530
Solna	1.50	29.80	29.27	0.0306	0.0945	0.3152
Stockholm	1.48	30.35	29.17	0.0515	0.0650	0.2877

Age	Births	Mid-year population	Observed fertility rate	Fitted fertility rate
14.5	12	164845	0.0001	0.0003
15.5	46	164512	0.0003	0.0011
16.5	166	162142	0.0010	0.0029
17.5	657	162440	0.0040	0.0066
18.5	1598	168120	0.0095	0.0131
19.5	3855	176112	0.0219	0.0231
20.5	7046	181306	0.0389	0.0369
21.5	10249	183334	0.0559	0.0540
22.5	13683	179586	0.0762	0.0733
23.5	16282	173338	0.0939	0.0931
24.5	18115	165463	0.1095	0.1114
25.5	20255	161876	0.1251	0.1263
26.5	21859	161162	0.1356	0.1364
27.5	22646	161932	0.1398	0.1410
28.5	23100	164772	0.1402	0.1401
29.5	22523	166848	0.1350	0.1343
30.5	21098	168399	0.1253	0.1244
31.5	18703	167567	0.1116	0.1118
32.5	16457	168646	0.0976	0.0976
33.5	14103	169725	0.0831	0.0830
34.5	11890	171756	0.0692	0.0689
35.5	9887	174184	0.0568	0.0558
36.5	8204	179698	0.0457	0.0443
37.5	6576	187785	0.0350	0.0345
38.5	4986	194073	0.0257	0.0263
39.5	3851	198248	0.0194	0.0198
40.5	2672	1 99 140	0.0134	0.0146
41.5	1777	198446	0.0090	0.0106
42.5	1114	193180	0.0058	0.0076
43.5	691	183102	0.0038	0.0054
44.5	344	168141	0.0020	0.0037
45.5	163	154129	0.0011	0.0026
46.5	61	146759	0.0004	0.0000
47.5	22	143439	0.0002	0.0000
48.5	3	140710	0.0000	0.0000
49.5	9	136160	0.0001	0.0000

Year	Â	ĉ	ƙ	â	10 ⁶ S
1950	2.3621	0.3022	5.0538	12.0478	145.33
1951	2.2680	0.3081	5.0813	12.0943	96.12
1952	2.2848	0.3013	4.7217	12.7970	81.86
1953	2.3039	0.3094	4.8385	12.6997	71.86
1954	2.2261	0.3023	4.6204	13.0407	77.81
1955	2.2820	0.3088	4.6068	13.2117	59.22
1956	2.3130	0.3374	5.2802	12.3676	62.00
1957	2.3036	0.3535	5.6528	11.7939	51.91
1958	2.2717	0.3669	5.8648	11.9205	31.60
1959	2.2508	0.3807	6.0891	11.7997	32.05
1960	2.1920	0.4008	6.5932	11.3181	16.92
1961	2.2316	0.4246	7.2683	10.553	18.07
1962	2.2653	0.4498	8.0474	9.6135	14.51
1963	2.3438	0.5163	10.2619	7.5202	16.41
1964	2.4955	0.5876	13.2017	4.8047	15.75
1965	2.4363	0.5874	13.2928	4.5254	17.74
1966	2.3928	0.6054	14.1886	3.5924	16.21
1967	2.3001	0.6116	14.0717	3.9529	17.82
1968	2.1038	0.6160	13.8407	4.5438	12.34
1969	1.9524	0.6152	13.1343	5.7213	5.00
1970	1.9468	0.5819	11.4473	7.3370	6.23
1971	1.9858	0.5832	11.1354	7.7903	5.41
1972	1.9341	0.6028	11.5695	7.6053	3.51
1973	1.8879	0.6083	11.5326	7.8697	3.03
1974	1.8975	0.6113	11.6804	7.6820	2.39
1975	1.7884	0.6638	13.3154	6.7353	1.81
1976	1.6963	0.6705	13.4701	6.8578	1.70
1977	1.6548	0.6595	12.8203	7.6267	1.69
1978	1.6083	0.6646	13.0885	7.6080	2.30
1979	1.6617	0.6953	14.1627	7.1844	6.49
1980	1.6878	0.6723	13.3482	7.8456	3.51
1981	1.6442	0.6919	14.2926	7.1963	4.01
1982	1.6275	0.7316	15.7802	6.4290	2.83
1983	1.6195	0.7499	16.3745	6.3058	2.78
1984	1.6604	0.8056	18.6724	5.1747	1.75
1985	1.7432	0.8531	20.9785	3.8604	1.65
1986	1.8043	0.8785	22.2372	3.1849	2.13

Table A4.-Estimated parameters and sum of squared deviations in fitting the gamma probability density function to observed five-year age-specific fertility rates for Sweden, 1950-86

N/	Ŕ	^ c	Â k	â	10 ⁴ S
Year	ĸ	С	K	a	10 5
1929	2.7278	0.2882	4.9132	13.0142	17.05
1930	2.6724	0.2914	4.9904	12.8854	17.19
1931	2.4387	0.2853	4.8139	13.1355	12.24
1932	2.2589	0.2971	5.1821	12.5594	10.70
1933	2.2325	0.3183	5.6933	12.0073	10.87
1934	2.1728	0.3059	5.2689	12.6671	9.22
1935	2.1744	0.3262	5.6535	12.3312	8.28
1936	2.2408	0.3236	5.5507	12.4793	8.22
1937	2.2681	0.3287	5.5326	12.6386	7.54
1938	2.2601	0.3349	5.6083	12.6570	6.36
1939	2.2754	0.3356	5.5304	12.8561	6.39
1940	2.3038	0.3399	5.4948	13.1301	6.04
1941	2.4178	0.3197	4.8526	14.0387	7.05
1942	2.4365	0.3048	4.5297	14.4547	8.32
1943	2.6351	0.3043	4.4606	14.6706	11.25
1944	2.7007	0.3308	5.3495	13.5261	15.34
1945	2.8041	0.3190	5.0379	13.9173	16.24
1946	3.0496	0.3057	4.4689	14.8589	15.79
1947	3.1350	0.2880	3.9943	15.2701	15.32
1948	3.0405	0.2933	4.1197	14.9557	12.41
1949	3.0250	0.2874	3.9079	15.2217	11.51
1950	3.1094	0.2956	4.0311	15.0829	10.82
1951	3.1072	0.2961	3.9985	15.0267	8.62
1952	3.2304	0.2886	3.7727	15.3520	10.03
1953	3.2412	0.2877	3.6898	15.5000	10.49
1954	3.2391	0.2929	3.7212	15. 49 87	10.45
1955	3.3185	0.2980	3.7381	15.4997	11.84
1956	3.3647	0.3191	4.1067	15.0000	13.50
1957	3.4705	0.3237	4.1475	15.0000	13.10
1958	3.4715	0.3096	3.8366	15.4960	9.60
1959	3.4901	0.3167	3.9349	15.3677	9.50
1960	3.5036	0.3204	3.9859	15.3361	9.20
1961	3.5895	0.3197	4.0024	15.2374	10.55
1962	3.4924	0.3287	4.2166	14.9516	8.55
1 963	3.3907	0.3414	4.5281	14.4859	8.66
1964	3.2144	0.3498	4.8670	13.8924	9.16
1965	3.0155	0.3708	5.2481	13.4770	9.70
1966	2.9047	0.3893	5.6656	12.9568	10.20
1967	2.8749	0.4085	6.0165	12.6670	8.90
1968	2.9096	0.4307	6.4659	12.2923	8.80
1969	2.8704	0.4527	7.0247	11.7286	7.20
1970	2.8343	0.4857	7.9776	10.7154	7.65

Table A5.-Estimated parameters and sum of squared deviations in fitting the gamma probability density function to age-specific fertility rates for Australia, 1929-70

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