

Computing elementary  
aggregates in the  
Swedish consumer price index

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COMPUTING ELEMENTARY AGGREGATES IN THE SWEDISH

CONSUMER PRICE INDEX

BY

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STATISTICS SWEDEN

Abstract: This report presents the philosophy and approach taken by Statistics Sweden in revising its computation methods for elementary aggregates in its Consumer Price Index. A distinction is made between a) the ideal definition, b) the operational parameter and c) the estimator of the elementary aggregate. Different alternatives for operational parameters are discussed with respect to their approximation properties and performance with respect to Fisher-type tests. The parameter which was adopted in the Swedish CPI in 1990 is presented; this parameter has not, as far as we know, been used earlier.



## 1. Introduction

In 1989-1990 some significant changes have taken place in the Swedish Consumer Price Index (CPI). A new computation method for elementary aggregate indices has been adopted, a new sampling and estimation system has been launched - see Ohlsson (1990) - and a new method of measuring prices and calculating indices for clothing items is scheduled for 1991.

This report deals with the first of these changes - the new parameter for elementary aggregates.

A CPI system consists of a hierarchy of price observations for items, combined into commodity groups at successively higher levels. At the highest level different commodities are weighted together, usually according to the Laspeyres' or a fixed quantity index, using information from e.g. household expenditure surveys (in Sweden also the National Accounts). At lower levels we usually have to manage with less relevant weighting information or simply use unweighted measures. The elementary aggregate is defined as the first level at which price observations are combined. Usually these prices are for the same item in different outlets (places where goods or services are sold), perhaps in a specific region or type of outlet. At this level it is not possible to apply the fixed quantity formula in a straightforward manner, since we lack the relevant weights.

In the last ten years or so a number of papers on this topic have emerged. See for example Forsyth (1978), Carruthers, Sellwood and Ward (1980), Morgan (1981), Szulc (1983), Turvey et al. (1989) and Szulc (1989).

In this paper we present the philosophy and approach to these problems, that has been adopted in the Swedish CPI. We believe that much of this philosophy might also be relevant for other countries.

First we give a brief introduction into the basic structure of the Swedish CPI in section 2. In section 3 we present an approach to the conceptual problems. In section 4 we discuss the definition and in section 5 the path from a definition to an operational parameter for an elementary aggregate of a CPI. In section 6 index tests for elementary aggregates are formulated and in section 7 the various alternatives for operational parameters which have been considered. In section 8 some further aspects of the parameter problem, relating to the two-dimensional nature of the population are discussed. In section 9 some mathematical relations between the parameters are given (most proofs are deferred to an Appendix). Finally, in section 10, data and specific circumstances in the Swedish CPI are further discussed.

## 2. Basic structure of the Swedish CPI

The Swedish CPI uses annual chains from december year  $t-1$  to december year  $t$ . We make a distinction between a long-term index,

which uses quantity weights  $q_t$  from year  $t$  and a short-term index which uses quantity weights  $q_{t-1}$  from year  $t-1$ . The long-term annual index is

$$I_{t-1,dec}^{t,dec} = \frac{\sum P_{t,dec} Q_t}{\sum P_{t-1,dec} Q_t}, \quad (1)$$

and the short-term index is

$$S_{t-1,dec}^{t,m} = \frac{\sum P_{t,m} Q_{t-1}}{\sum P_{t-1,dec} Q_{t-1}}, \quad (2)$$

where as usual  $p_{t,m}$  is the price for month  $m$  of year  $t$  and  $\sum$  stands for summation "over all commodities" (see Section 4 below) bought by private consumers.

The chained index from december year 0 to month  $t,m$  then becomes

$$I_{0,dec}^{t,m} = S_{t-1,dec}^{t,m} \prod_{j=1}^{t-1} L_{j-1,dec}^{j,dec}, \quad (3)$$

Note that, in the long run, the Swedish CPI is only dependent on the long-term indices, since the short-term indices are successively replaced by their long-term counterparts. This system - used in Sweden since the middle of the 1950s - is actually a rather ingenious way of avoiding the well-known Laspeyres' upward and Paasche downward bias, since the weights, estimated by the National Accounts, represent the period between the base and the reference period. To the best of our knowledge no other country uses a similar procedure.

Both the long-term and the short-term indices are computed as weighted commodity indices ( $I$ ), where the weights ( $w$ ) are revalued, commodity by commodity, to the correct reference month (december year  $t-1$ ). (1) and (2) could be rewritten as:

$$CPI = \sum w I, \text{ with } w = \frac{Q_b P_{t-1,dec}}{\sum Q_b P_{t-1,dec}} \text{ and } I = \frac{P_{t,m}}{P_{t-1,dec}} \quad (4)$$

where  $b$  is the weight base period ( $t$  or  $t-1$ ) and  $m$  is the reference month of the index. This applies both to the long-term and the short-term index.

The commodity indices  $I$  are defined and computed in various ways, depending on the kind of price and weight information available in different cases. In this paper we primarily discuss commodities in the LOCAL PRICE SYSTEM (LOPS) and the LIST PRICE SYSTEM (LIPS). These systems together account for about 46% of the total weight of

the Swedish CPI.

In LOPS, which covers clothing items, fresh food, furniture, household appliances etc., prices are collected by field interviewers each month. As weights we use the size of the outlet in terms of number of employees.

In LIPS, which covers most food items and other daily necessities, prices are taken from price lists (except for december each year). Here we use as weights, besides the size of the outlet, also the total sales of a certain item.

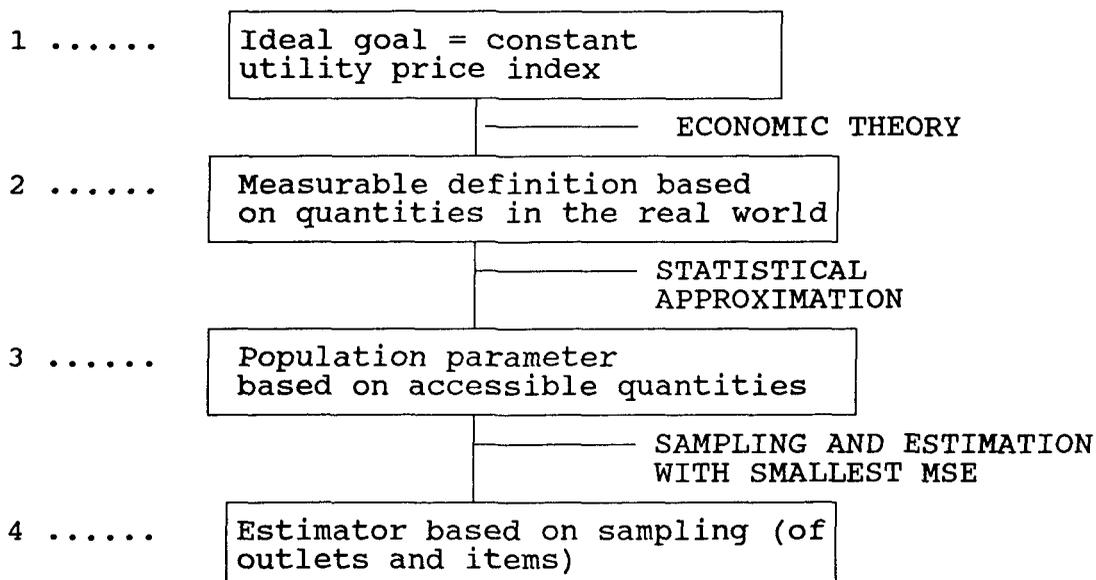
Both in LOPS and in LIPS the weights are in practice from an earlier period than b in (4) above. In both systems there is a random (PPS=probability proportional to size) sampling of outlets stratified by industry (LOPS) or by region (LIPS). In LIPS there is also a random (PPS) sampling of items within item groups but in LOPS the items are purposively selected.

### 3. An approach to the conceptual problems in the CPI

In a statistical survey in general we can formulate the trilogy: Ideal Goal - Statistical Goal - Estimate from a Sample.

Here the ideal goal relates to the underlying problem of the users of the survey. The statistical goal is the ideal goal translated into the statistical world of populations and measurable quantities and is a property defined for an entire target population. Finally, from this population a sample is drawn and an estimate is made.

For a CPI the following scheme, divided into four steps would provide an analogy:



The first step in this scheme is somewhat controversial, for several reasons. Firstly there is not, even theoretically, any such thing as a constant utility price index (or, as it is also called, a cost-of-living index) defined for a group of people, let alone the whole population of a country. Allen (1975) calls it "an act of faith" to go from the constant utility price index for an individual to an aggregate, social price index. Later, theoretical attempts in this direction have been made, however, for example by Jorgenson and Slesnick (1983) and by Blackorby and Donaldson (1983). One issue here is democratic vs plutocratic index, which refers to whether individuals should be equi-weighted or weighted on the basis of their consumption. (Practical index construction usually approximates a plutocratic index.)

Secondly, there seems to be different opinions as to the value of basing official price index calculations on this theoretical concept at all. For example in ILO (1987) - a resolution concerning consumer price indices adopted by the Fourteenth International Conference of Labour Statisticians - no reference to the cost-of-living concept is made. Instead, there is the following statement of intent (page 124):

" The purpose of a consumer price index is to measure changes over time in the general level of prices of goods and services that a reference population acquire, use or pay for consumption."

On the other hand BLS (1988), the official handbook of methods for the U.S. CPI, explicitly refers to the concept of the cost-of-living index as

"a unifying framework for dealing with practical questions that arise in construction of the CPI".

We basically agree with this latter point of view since it is necessary to have a theoretical point of departure in situations of quality change and changes in the sets of items and outlets, that we measure. However, references to the cost-of-living index have to be mainly intuitive.

The second step in this scheme refers to a concept in which all components are in principle observable in the physical world (unlike the constant utility price index, where utility is unobservable). This definition should be in terms of detailed data (prices and quantities) on all transactions taking place on the market and must go all the way from a price observation for an item in an outlet up to the all items CPI. Essentially this is a task for economic research, where more efforts should be put in.

The third step, the population parameter, refers to a concept based on those quantities that are accessible in the statistical practice with budget restrictions etc. These quantities are considerably less detailed and in some respects not quite up-to-date. Typically we do not have relevant quantity measures to attach to single price

observations but we may have more crude size measures like total sales of an outlet or an item. This statistical parameter should ideally be arrived at through a series of approximations from a measurable definition in step 2 or, perhaps, directly from the basic definition in step 1. Since, as we will see, such an approximation philosophy does not lead to a definite answer there is also a need for an alternative approach, the so called axiomatic approach, based on index tests.

The fourth step is the result of a traditional statistical sampling procedure. Probability sampling is of course the preferred technique but sometimes frame problems and the necessity of obtaining comparable price observations at two time periods makes purposive, balanced samples the only possibility, particularly in the item dimension. Based on some cost restriction, the variance or the mean square error of the estimator (in relation to the population parameter) should be minimized.

In summary we look at the constant utility price index as the Ideal Goal of the CPI in spite of the difficulties in defining this concept for a group of people. It is not, however, possible to clearly point at one well-defined Statistical Goal. Instead, this goal will differ between different commodities for reasons that have to do with the type of consumption involved as well as with the kind of data available.

#### 4. The definition of a CPI

##### 4.1 The basic definition

The basic definition of almost any CPI like (1) and (2) above is of the fixed-quantity type:

$$\text{CPI}_{0t} = \frac{\sum P_t Q_b}{\sum P_0 Q_b}, \quad (5)$$

where summation is over commodities or commodity groups,  $b$  is the expenditure base period,  $0$  is the reference period and  $t$  is the comparison period. At some points in time (in Sweden annually) the expenditure base is updated and indices are chained. If  $b < 0$ , this definition is generally believed to be an over-estimate of a constant utility price index due to the consumer's possibility to move from commodities for which the prices increase more to those for which they increase less or decrease.

This is really only a beginning of a CPI definition and leaves many problems unsolved, for example:

1) It presupposes an all-inclusive commodity grouping related to the expenditure base period. It does not say what to do below that level, for which we have relevant weights.

2) It does not deal with the problem of quality change, that is what to do when we do not longer have an item identical to that in period b. Different practices have evolved here, like different variants of linking or imputation, direct quality evaluation or hedonic regression. None of them could be motivated by definition (5) but only by reference to the underlying concept of the cost-of-living index.

3) It does not deal with the problem of new or disappearing items. There is a need for some theoretic guidance for example as to when an item should be considered a new commodity resp. a new variant of an old commodity with, perhaps, a new quality. This question is, in a sense, impossible for economic theory proper, since the cost-of-living index is not generally even separable into different indices for different commodities.

4) It does not deal with the problem of new or disappearing outlets. If, for example a new discount store opens (or closes), its prices are, in standard CPI practice, not compared with those of the old outlets with the result that its direct effect on the price level is not estimated. Theoretically there could not be a good motive for this practice, even though one may consider the new store to have a different service level and thereby also a different quality than the old ones. The risk for systematic errors is still considerable with the present practice.

5) It does not say how to handle items for which the price changes within the reference/comparison period in the same outlet. Here there is usually a purposive sampling in time of one day in the period, in Sweden a day in the middle week of a month. But theoretically, are we supposed to measure a monthly average price or what?

In all these five respects there is neither a theoretical nor a practical consensus in practical index making of what to do.

#### 4.2 The definition of an elementary aggregate

You could look at equation (5) in two ways, dividing the basic definition into two variants.

**DEFINITION 5A:** We think of the summation as applying only to some basic division of private consumption into elementary aggregates (which may be item groups like milk, newspapers, haircuts etc.). For one such elementary aggregate we interpret the prices -  $P_t$  and  $P_0$  - as mean prices. These are defined as quantity weighted arithmetic means of all prices paid for all items within the group, in all outlets and during all days in the period (usually a month).

A shortcoming of this definition is that it leaves quality differences outside - either they have to be taken care of in some other way or you have to presuppose that the aggregates are

homogeneous in quality. It does not seem to be applied much in practice. It would require another sampling philosophy than the one usually applied with, e.g., separate item and outlet samples for each time period. On the other hand, at least presupposing constant quality, this definition probably is a better approximation to a constant utility price index than definition 5B below.

DEFINITION 5B: We think of the summation as being over all item/outlet pairs in period b. This is for example the viewpoint of Gillingham (1974). P and Q are then interpreted to be quantities at the micro level.

This is in a way a very consistent definition but it has many shortcomings. Firstly it does not take care of varying prices for the same item in the same outlet during the reference period - you would still have to define a mean price within that period in some way (or consider 0 and t to be fixed points in time rather than periods which would give you a less relevant index). Secondly it is not possible to apply 5B in practice unless b is sufficiently far before 0. Thirdly, given that b is far before 0, 5B accentuates the tendency of a fixed quantity index to overestimate a constant utility price index since it does not even take into consideration the possibility for a consumer to change to a closely related but cheaper item or to a nearby outlet. Fourthly, the problem of linking (or not linking) new items and outlets to old ones remains.

Neither of these definitions could be applied exactly in practice. We lack information for constructing an item/ outlet matrix with information on what outlets sell what items in what quantities. An interesting future perspective, however, is the introduction of computerized cash desks in many stores. If traded quantities are registered this way, it would enable us to work with more sophisticated definitions in our price indices!

##### 5. From definition to parameter

For reasons discussed above we have to base our index calculations for elementary aggregates on some simple averaging formula. Therefore we have to find a rationale for our choice of this formula.

Just like in any survey we make a distinction between the population parameter and the estimator. The population parameter is considered to be the quantity to be estimated, or what we had obtained if we had been able to measure all outlets (and in LIPS also all items) in the population. In the case of price indices the parameter is equivalent to the basic index formula. The estimator is then determined using sampling theory and depends on the sampling design used (in our case PPS).

For elementary aggregates we typically do not have any useful quantity weights. Instead we have crude size measures related to

the total sales of an outlet or no weights at all. (Some writers - e.g. Allen (1975) - refer to indices without quantity weights as "stochastic".) In the Swedish CPI we use size measures which are related to total sales (that is price\*quantity) of an outlet or an item in a certain time period.

Let us define a price index in this context as a function P of two price vectors and one weight vector.

$$\begin{aligned}
 P &= P(\vec{w}, \vec{p}_t, \vec{p}_0), \text{ where} \\
 \vec{w} &= (w_1, \dots, w_N), \\
 \vec{p}_t &= (p_{t1}, \dots, p_{tN}) \text{ and} \\
 \vec{p}_0 &= (p_{01}, \dots, p_{0N})
 \end{aligned} \tag{6}$$

Now, our first choice for finding a reasonable parameter is trying to approximate a definition like 5A or 5B in section 4 above.

Starting from definition 5A above the index could be expressed as a ratio of mean prices weighted by the quantity q bought at each price. This makes it possible to pursue the following series of approximations. The first of these is the fact that an average price is, in the absence of the relevant quantity weights, best computed as a harmonic mean. This point is further explained in Ohlsson (1989).

$$\begin{aligned}
 \frac{P_t}{P_0} &= \frac{\bar{p}_t}{\bar{p}_0} = \frac{\Sigma q_t p_t / \Sigma q_t}{\Sigma q_0 p_0 / \Sigma q_0} \approx [w \approx pq] \approx \\
 &\approx \frac{\Sigma w_t / \Sigma w_t / p_t}{\Sigma w_0 / \Sigma w_0 / p_0} = \frac{\Sigma \frac{w_0}{\Sigma w_0} / p_0}{\Sigma \frac{w_t}{\Sigma w_t} / p_t} = H_A \approx
 \end{aligned} \tag{7}$$

$$\approx \left[ \begin{array}{l} \text{if the population of outlets} \\ \text{is unchanged and the weight} \\ \text{structures are similar} \end{array} \right] \approx \frac{\Sigma' w / p_0}{\Sigma' w / p_t} = H$$

Here we have two levels of approximation -  $H_A$  and H.  $H_A$  is closer to definition 5A and applying it means having different sets of weights and outlets in the numerator and the denominator. In H you further simplify by using the same weights and outlets. This second approximation is probably too crude for H to be of any practical use. On the other hand; for commodities that are homogeneous in quality  $H_A$  may well be a good approximation to definition 5A and a constant utility price index.

On the other hand starting from definition 5B above the fixed weight principle is considered applicable down to the very lowest level of an item/outlet pair. In this case we could use the following series of approximations.

$$\frac{\sum P_t Q_B}{\sum P_0 Q_B} = \frac{\sum P_B Q_B (P_t/P_B)}{\sum P_B Q_B (P_0/P_B)} \approx$$

$$\approx [W_B \approx P_B Q_B] \approx \frac{\sum W_B (P_t/P_B)}{\sum W_B (P_0/P_B)} \quad (8)$$

Now, in the Swedish long-term index,  $P_B$  is the average price over the period from 0 to  $t$ . It can be approximated by, e.g.,  $P_B = (P_0 + P_t)/2$  (another, slightly better but less practical, approximation would be  $P_B = (P_{\text{Jan}} + P_{\text{Feb}} + \dots + P_{\text{Dec}})/12$ ) leading to

$$RA = \frac{\sum W_B P_t / (P_0 + P_t)}{\sum W_B P_0 / (P_0 + P_t)} \quad (9)$$

Using definition 5B we thus arrive at another operational parameter, called RA, after a series of suitable approximations. This approximation is especially geared to the type of base period used in the definition of the Swedish long-term-index.

## 6 Index tests

Since the times of Irving Fisher (1922) one strand of index theory - the so-called axiomatic approach - has been to formulate certain tests (or axioms) that the various index formulae should pass. A later paper in this tradition is Eichhorn and Voeller (1983). They define a price index to be a mathematical function of 2 x 2 vectors of prices and quantities at the two time periods with  $N$  elements each. This function is to fulfill certain tests of a completely mathematical nature.

For elementary aggregates there is one fundamental difference compared with the Eichhorn and Voeller system: no quantities are available. We formulate the following six tests, considering them to be the most essential ones and relate their properties to the Eichhorn and Voeller system.

### I Monotonicity Test

The function  $P$  is strictly increasing with respect to  $p_t$  and decreasing with respect to  $p_0$ :

$$P(\vec{w}, \vec{p}_t, \vec{p}_0) > P(\vec{w}, \vec{p}'_t, \vec{p}_0) \text{ if } \vec{p}_t \geq \vec{p}'_t \text{ and}$$

$$P(\vec{w}, \vec{p}_t, \vec{p}_0) < P(\vec{w}, \vec{p}_t, \vec{p}'_0) \text{ if } \vec{p}_0 \geq \vec{p}'_0 \quad (9)$$

Interpretation: if at least one price ratio increases (decreases) while the others remain equal then the price index also increases (decreases). (The sign  $\geq$  means that all arguments are larger and at

least one argument is strictly larger.)

## II Proportionality Test

If all corresponding prices differ by the same factor  $c$ , then the value of  $P$  equals  $c$ :

$$P(\vec{w}, c\vec{p}_0, \vec{p}_0) = c \quad (10)$$

## III Price Dimensionality Test

The same proportional change in the unit of the currency does not change the value of  $P$ :

$$P(\vec{w}, c\vec{p}_t, c\vec{p}_0) = P(\vec{w}, \vec{p}_t, \vec{p}_0) \quad (11)$$

This test may appear to do the same thing as the Proportionality Test. However, Eichhorn and Voeller (1983, page 420) give an example of a function passing II but not III, which also applies to our elementary aggregate situation. III is, however, a special case of IV below.

## IV Change-of-Units Test

The same change in the units of measurement of the corresponding items does not change the value of  $P$ .

$$P(\vec{w}, c_1 p_{t1}, \dots, c_N p_{tN}, c_1 p_{01}, \dots, c_N p_{0N}) = P(\vec{w}, \vec{p}_t, \vec{p}_0) \quad (12)$$

This test gets a different formulation in the absence of quantity weights. The test is fulfilled by all functions that only use the price ratios and not the prices themselves.

## V (Price/Time) Reversal Test

Eichhorn and Voeller (1983) formulate two reversal tests: the Time Reversal and the Price Reversal Tests. These two tests coincide in the absence of quantity weights. We get the following formulation of the Reversal Test:

$$P(\vec{w}, \vec{p}_t, \vec{p}_0) P(\vec{w}, \vec{p}_0, \vec{p}_t) = 1 \quad (13)$$

This means that the price index calculated backwards from  $t$  to 0 with the same weights as the forwards index should be the inverse of the forwards index.

## VI Permutation (Price Bouncing) Test

*If all  $w_j = 1/N$  and if*

*$\vec{p}_t$  and  $\vec{p}_0$  are arbitrary permutations*

*of  $\vec{p}_t$  and  $\vec{p}_0$  respectively then*

(14)

$$P(\vec{w}, \vec{p}_t, \vec{p}_0) = P(\vec{w}, \vec{p}_t, \vec{p}_0)$$

This very strong test has, as far as we know, not been proposed earlier. The practical need of a test like this stems from the "price bouncing" behaviour of the outlets (see section 10 below). A special case of this test is the following:

VI-B: *If all  $w_j = 1/N$  and if*

*$\vec{p}_t$  is a permutation of*

*$\vec{p}_0$  then  $P(\vec{w}, \vec{p}_t, \vec{p}_0) = 1$*

(15)

This is a special case if also the proportionality test is fulfilled with  $c=1$ . Perhaps this special case is enough for practical purposes.

What VI-B says is that the price index should show no change if prices "bounce" in such a manner that the outlets are just exchanging prices with each other. This could, for example, be the result of outlets moving up and down from ordinary to campaign prices so that there is the same set of prices in the market in both time periods but in different outlets. If all weights are equal, it is intuitively obvious that the price level has not changed in this case.

## 7 Operational parameters for elementary aggregates

In this section we assume that there is only one summation level - over outlets for a specific item like in the Swedish LOPS. The parameters are easily generalized to cases where the outlets are stratified or clustered. The case of summation also over items in an item group is discussed in the next section.

The weights are normed so that  $\sum w = 1$ . For those parameters where we have more than one summation, all summations are over the same set of outlets.

## 7.1 The ratio of mean prices

This parameter (labelled A) has the following form:

$$A_{0t} = \frac{\sum w p_t}{\sum w p_0} \quad (16)$$

It is simple to understand and seems to be the most popular elementary aggregate formula on a global scale. It was used in the Swedish CPI up to 1989.

Still it has one considerable shortcoming. If the weights  $w$  are related to sales amounts, then  $w \approx p_b q_b$ , where  $b$  is a weight base period and

$$A_{0t} \approx \frac{\sum q_b p_b p_t}{\sum q_b p_b p_0} \quad (17)$$

This means that - compared with the basic definition (5) - there is a weighting with  $p_b$  in addition to the quantity weight. This is absurd and means that in effect  $A$  overweights the more expensive items and underweights the cheaper ones. This leads to an index relevant for luxury consumers!

From the point of view of the above index tests  $A$  fulfills all of them except the Change-of-units Test. In practice this means that, if for example one outlet sells eggs per dozen instead of per kg and the price is rescaled to the other unit, this would change the resulting index number if  $A$  is used.

The shortcomings of  $A$  must be judged to be serious. It must be noted, however, that  $A$  is the natural choice if  $w$  were quantity weights. If these weights also represent the weight base period  $b$ , the  $A$  parameter is equivalent to the basic CPI definition (5), which meets the Change-of-units Test.

## 7.2 The mean of price ratios (Sauerbeck index)

This parameter (labelled  $R$ ) has the following form:

$$R_{0t} = \sum w \frac{p_t}{p_0} \quad (18)$$

$R$  is also simple to understand and widely used. It was, unfortunately, used in the Swedish CPI (short-term index) from January to March 1990. It does not have the problem of the  $A$  parameter - low and high prices influence the index to the same extent and consequently it meets the Change-of-units Test. Still it was abandoned from April 1990 and onwards. Why?

$R$  passes all of the above index tests except the Reversal Test and the Permutation Test. It is an easy mathematical exercise (see section 8) to show that

$$R_{0t} \times R_{t0} \geq 1 \quad (19)$$

It is likewise possible (see 9.1.2 and Appendix) to show that if the prices at time  $t$  are a permutation of those at time 0 then

$$R_{0t} \geq 1 \quad (20)$$

In both (19) and (20) equality only applies if all prices are equal at 0 and t. These two relations show that R has an upward bias, which could be quite large in practice (see section 10) due to so called price bouncing. Price bouncing is a frequent phenomenon in Sweden and probably also in many other countries - see e.g. Szulc (1989, page 175). It occurs when prices for an item move up and down from month to month due to seasonal variation, campaign prices or substitutions. The movements could be tens and in extreme cases even hundreds of percent.

A simple example illustrates the effect. Say that the regular price of coffee is 25 SKr/½ kg. But coffee is a commodity that is often subject to a campaign price of, say, 20 SKr/½ kg. Now suppose that one store reduces its price from period 0 to period t in this way. This means a price ratio of 20/25=0.8. Another store with the same weight goes the other way and raises its price which gives a price ratio of 25/20=1.25. Averaging these price ratios gives (0.8+1.25)/2=1.025 - a price increase by 2.5%! Since the price level is unchanged this is absurd. See section 10 for further illustrations.

### 7.3 The geometric mean

This parameter (labelled G) has the following form:

$$G_{0t} = \pi (p_t/p_0)^w = \frac{\pi (p_t)^w}{\pi (p_0)^w} \quad (21)$$

By looking at these alternative expressions one of its advantages is immediately seen - by taking geometric means you have an equivalence between means of ratios and ratios of means.

G also meets all of the above index tests - the Change-of-units Test (which A fails) as well as the Reversal Test and the Permutation Test (which R fails).

It is an open question to what extent G could be motivated by economic theory. Pollak (1983) formulates conditions under which the cost-of-living index (COL) is a geometric mean. This depends on the form of the utility function and whether the COL is a function of price relatives only (and not prices in some other combination). In CPI practice, however, we take the basic definition (5) for granted and the issue of geometric means only applies to the elementary aggregate level. It is not possible to find a simple approximation method leading from (5) to G.

In a resolution of the Fourteenth International Conference of Labour Statisticians (ILO (1987)) there is, however, a specific recommendation:

"In the calculation of elementary aggregate indices, consideration should be given to the possible use of geometric means."

#### 7.4 The ratio of harmonic means

This parameter (labelled H) has the following form:

$$H_{0t} = \frac{\sum w/p_0}{\sum w/p_t} \quad (22)$$

As described in section 5 above this parameter is arrived at by approximating definition 5A above.

However, alike A, H does not meet the Change-of-units Test. But contrary to A, H actually gives larger weight to relatively less expensive items. This could be seen by rewriting it as

$$H = \sum (w/p_t) (p_t/p_0) / \sum (w/p_t) \quad (23)$$

It thus becomes some kind of poor man's index, which may seem more attractive than the contrary. Still we consider it to be a shortcoming, since the w reflect all weight information that we want to use.

#### 7.5 The harmonic mean of ratios

This parameter has the following formula:

$$H'_{0t} = \frac{1}{\sum wp_0/p_t} \quad (24)$$

It has the inverse properties of the R parameter. It passes all tests except for the Reversal Test and the Permutation Test. By observing that  $H'_{0t} = 1/R_{t0}$  we immediately obtain that

$$H'_{0t} H'_{t0} < 1, \quad (25)$$

which means that it has a negative bias and underestimates a true index. It therefore has no interest in itself.

#### 7.6 The squared root of R and H'

The inverse properties of R and H' leads one to consider a combination of them. The natural combination is their geometric mean with the following formula:

$$RH_{0t} = \frac{(\sum wp_t/p_0)^{\frac{1}{2}}}{(\sum wp_0/p_t)^{\frac{1}{2}}} \quad (26)$$

RH passes all the above index tests except for the Permutation Test. However, it has no known theoretical motivation other than that.

#### 7.7 The ratio of normed mean prices

The last parameter to be discussed here is, as far as we know, a

novel invention. We call it RA and its formula is:

$$RA_{0t} = \frac{\sum w p_t/p.}{\sum w p_0/p.}, \text{ where } p. = (p_0 + p_t)/2. \quad (27)$$

The arguments for this parameter follow two lines. Firstly, as described in section 5 above this parameter is arrived at by approximating definition 5B above.

Secondly, it passes all the above index tests except the Permutation Test. But unlike R it can be demonstrated that it has no definite upward or downward bias; RA could be larger as well as smaller than or equal to 1 in a permutation situation and differences are always small. We may therefore, somewhat loosely, state that RA approximately passes the Permutation Test.

**Thus RA is the only parameter which has strong support by index tests as well as approximation arguments.**

## 8 The two-dimensional weight structure

In the parameter formulations in section 7 above we have assumed that we have just one type of weight for every price observation. But reality is more complicated than that!

For many commodities or item groups there could be several weight/summation levels. However, two dimensions are basic: the item dimension and the outlet dimension.

So, now we consider a two-dimensional division of a certain item group according to consumption in the weight base period. In the vertical dimension we divide the item group into (homogeneous) items and in the horizontal dimension we have all outlets selling (in principle at least one of) the items in the item group. Ideally we now would like to have the complete two-dimensional weight structure, showing how much outlet  $j$  sold of item  $k$ . In practice many of these weights would be zero. This information is, at least in Sweden, not available. The weights that are available are marginal weights giving more or less good approximations to the relative sales of item  $k$  or of outlet  $j$ .

In addition to that, we are able to observe the zero weights in the sample, i.e. the outlets which do not trade item  $k$  (see  $l_{jk}$  below). We have to take account of this additional information since an outlet with no price for item  $k$  automatically gets a zero weight for that item.

Let us use the following symbols:

$w_k$  is the weight of item  $k$ ,

$v_j$  is the weight of outlet  $j$ ,

$l_{jk}$  (an indicator) is 1 if item  $k$  is sold in outlet  $j$  and 0 otherwise.

(Theoretically the weights should be related to the base period  $b$ . The indicators should be 1 only if the item is sold in all three time periods  $b$ , 0 and  $t$ . But in practice the weights will refer to different time periods and there will be item substitutions which further obscure the matter.)

Because of this two-dimensional nature of the population, each parameter now divides into three "subparameters".

Let us take  $RA$  as an example. If we had the full weight matrix  $\{w_{kj}\}$  we would have:

$$RA = \frac{\sum_k \sum_j w_{kj} P_{1kj} / P_{.kj}}{\sum_k \sum_j w_{kj} P_{0kj} / P_{.kj}} \quad (28)$$

When we approximate with marginal weights we have three choices:

$$RA_1 = \frac{\sum_k w_k \{(\sum_j v_j l_{kj} P_{tkj} / P_{.kj}) / (\sum_j v_j l_{kj})\}}{\sum_k w_k \{(\sum_j v_j l_{kj} P_{0kj} / P_{.kj}) / (\sum_j v_j l_{kj})\}} \quad (29)$$

$$RA_2 = \frac{\sum_j v_j \{(\sum_k w_k l_{kj} P_{tkj} / P_{.kj}) / (\sum_k w_k l_{kj})\}}{\sum_j v_j \{(\sum_k w_k l_{kj} P_{0kj} / P_{.kj}) / (\sum_k w_k l_{kj})\}} \quad (30)$$

$$RA_3 = \frac{(\sum_k \sum_j w_k v_j l_{kj} P_{tkj} / P_{.kj}) / (\sum_k \sum_j w_k v_j l_{kj})}{(\sum_k \sum_j w_k v_j l_{kj} P_{0kj} / P_{.kj}) / (\sum_k \sum_j w_k v_j l_{kj})} \quad (31)$$

(Note that in  $RA_3$  but not in  $RA_1$  or  $RA_2$  a reduction of the fraction is possible.)

The difference between these alternatives can be illustrated by the following example with three items and three outlets. The  $w_k$  are 0.4, 0.4 and 0.2 and the  $v_j$  are 0.6, 0.3 and 0.1 respectively and cells with  $l_{jk} = 0$  are marked with "-". Here we show the three weight structures that follow implicitly from using the above three alternatives.

RA <sub>1</sub>				RA <sub>2</sub>				RA <sub>3</sub>			
			Sum				Sum				Sum
.24	.12	.04	.40	.24	.20	.10	.54	.29	.15	.05	.49
.40	-	-	.40	.24	-	-	.24	.29	-	-	.29
.13	.07	-	.20	.12	.10	-	.22	.15	.07	-	.22
.77	.19	.04	1	.60	.30	.10	1	.73	.22	.05	1

We observe that in  $RA_1$  the item weights are preserved and the same is true for the outlet weights in  $RA_2$ . In  $RA_3$ , however, the weights

are not preserved in any dimension.

Therefore, the choice should usually be between  $RA_1$  and  $RA_2$ . The choice between those depends on what weights are the more important to preserve. For example, stratification (in the Swedish LIPS we stratify outlets by region) may call for preserving the weights in the stratification dimension.

This two-dimensional nature of the population also gives rise to important sampling, estimation and allocation problems, which are yet to be examined. Some contributions are given in Vos (1964).

## 9 Mathematical and statistical properties of the parameters

There are a number of important relations between the parameters above. From a mathematical point of view we note that  $R$ ,  $G$ ,  $H'$ ,  $RH$  and  $RA$  are all functions of the price ratios ( $p_t/p_0$ ) alone, while  $A$  and  $H$  depend also on the price levels  $p_t$  and  $p_0$  separately.

### 9.1 Inequalities

9.1.1.  $R_{0t} \times R_{t0} \geq 1$  with equality if and only if all price ratios are equal.

Proof:  $R_{0t} \times R_{t0} = \sum w_j p_{tj}/p_{0j} \times \sum w_j p_{0j}/p_{tj} \geq \sum w_j = 1$   
by the Cauchy-Schwarz inequality. Equality occurs when all  $p_{tj}/p_{0j}$ , which are the price ratios, are equal.

9.1.2 If  $\bar{p}_t$  is a permutation of  $\bar{p}_0$  and all  $w_j=1/N$  then  $R_{0t} \geq 1$ .

Proof: See Appendix.

9.1.3.  $H' \leq G \leq R$  with equalities if and only if all price ratios are equal.

Proof: These inequalities are simply the well-known relations between the arithmetic, geometric and harmonic means of positive real numbers.

9.1.4.  $H' \leq RH \leq R$  with equalities if and only if all price ratios are equal.

Proof: This follows from 9.1.3 and the definition of  $RH$ , which is  $RH = (H'R)^{\frac{1}{2}}$ .

9.1.5.  $H' \leq RA \leq R$  with equalities if and only if all price ratios are equal.

Proof: See Appendix. After some algebraic manipulations the Cauchy-Schwarz inequality is used.

Comment to 9.1.1 - 9.1.5: Note that these inequalities are strictly mathematical, only using the fact that price ratios are positive real numbers.

## 9.2 Approximations of differences between parameters

By making Taylor expansions around the unit vector (all  $r_j=1$ ) it is possible to establish approximate relations between those parameters, that are functions of price ratios only. It turns out that these relations could be expressed in terms of the following weighted moments of the price ratios  $r_j = p_{tj}/p_{0j}$ :

$$\begin{aligned} \mu &= \sum w_j r_j \\ \sigma^2 &= \sum w_j (r_j - \mu)^2 \text{ and} \\ \gamma &= \sum w_j (r_j - \mu)^3 \end{aligned} \quad (32)$$

The accuracy of these approximations depends of course on how far away from one the price ratios are, but in most practical situations they could be considered quite reliable. The derivation of these relations are sketched in the Appendix.

The relations are:

9.2.1  $R - G \approx R - RA \approx R - RH \approx \sigma^2/2$  as a result of second-order Taylor approximations.

9.2.2  $R - H' \approx \sigma^2$  as a result of second-order Taylor approximations.

9.2.3.  $G - RA \approx \gamma/12 + (\mu-1)\sigma^2/4$  as a result of third-order Taylor approximations.

Comment to 9.2.1-9.2.3. These approximations show the order of the positive bias of the parameter R in relation to the more accurate parameters G, RA and RH. As could be expected this bias increases with increasing dispersion of the price ratios. 9.2.3 means that G could be expected to be slightly larger than RA in situations of price increases with a positively skewed distribution. This is likely to be a more common case than the contrary.

## 10 Outcomes of the Swedish CPI with different parameters

For January-March and for September 1990 experimental calculations with different parameters were done for the Swedish CPI. Only LOPS and LIPS were involved so the choice of parameter only influenced about 45% of the CPI total. The results for September are presented in table 1. (They are typical also for the other months.)

The major feature of these results is the size of the differences, which runs contrary to the popular belief that the formula question

does not matter very much. With the R parameter the Swedish CPI figure for September 1990 as compared with September 1989 would have been 0.65 percentage units higher than the one presented - 110.63 instead of 109.98. The reasons for these considerable differences are the large variances of the price ratios or in a subject-matter oriented language the price bouncing phenomenon. Price bouncing has three major causes in the Swedish CPI data:

- Seasonal and other variations in the prices of fresh fruits and vegetables.
- Campaign pricing for short periods for many daily commodities.
- Substitutions caused by an item disappearing from the market. This is particularly common among clothing items.

Different commodity groups exhibit different patterns. The parameter differences are largest for fresh food and clothing. The most extreme price bouncing commodity was white cabbage, for which the price ratios one month ranged from 0.23 to 7!

The A parameter was used in the Swedish CPI up to 1989. This year a decision was taken to use the RA parameter in the long-term index and R in the short-term index. The reason for this decision was that RA could be motivated by approximation arguments in the long-term index only. The simplicity of R was the decisive factor in the short-term index since its bias effects were at that time thought to be negligible. The knowledge of the large differences exhibited in table 1 led to a change of this decision in May 1990 and from that time on the RA parameter is used in the short-term index as well.

Thus it can be said that the decision to apply the RA parameter in the Swedish CPI is motivated by approximation arguments as well as by its performance with regard to index tests. These arguments were judged to outweigh its more complicated structure, which makes it more difficult to explain to the general public.

## 11 Acknowledgement

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Table 1. Outcome of the Swedish CPI September 1990 according to different parameters in LOPS and LIPS.

Commodity group	Parameter					Weight (%)
	A	R	G	RA	H	
Fresh food	112.62	115.82	112.13	111.92	111.48	3.7
Clothing and shoes	101.01	103.71	100.94	100.98	100.89	7.6
Furniture and household equipment	104.94	105.96	104.68	104.68	104.57	5.2
Articles for recreation	102.97	104.31	102.93	102.93	102.78	4.2
Miscellaneous	112.17	113.29	112.47	112.42	112.75	6.5
LOPS, all items	106.30	108.15	106.23	106.20	106.15	27.4
LIPS, all items	107.02	107.49	106.85	106.85	(105.02)*	18.3
LIPS + LOPS	106.59	107.89	106.48	106.46	(105.70)*	45.7
Difference compared to RA for CPI, all items	+ 0.06	+ 0.65	+ 0.01	0	(- 0.35)*	

\* The low value is due to a few misprinted items.

## APPENDIX

Proof of 9.1.2:

We use a theorem in Hardy et al (1934, page 261) saying that, if  $\{a_j\}$  and  $\{b_j\}$  are two sets of  $N$  real numbers, which are given except in order (arrangement) then  $\sum ab$  is largest when the sets are ordered in the same direction (either both are increasing or both are decreasing) and smallest when the sets are ordered in the opposite direction. Now, setting  $\{b_j\} = \{1/a_j\}$  and letting  $\{i_j\}$  denote an arbitrary reordering of the elements in the original set, we get

$$\sum_{j=1}^N a_j \frac{1}{a_{i_j}} \geq \sum_{j=1}^N a_j \frac{1}{a_j} = N \quad (33)$$

where  $\{a_j\}$  and  $\{1/a_j\}$  are arranged in opposite direction on the right hand side of the inequality.

Now, setting  $a_j = p_{tj}$  and  $a_{i_j} = p_{0j}$  the theorem is proved.

Proof of 9.1.5

Setting  $r_j = p_{tj}/p_{0j}$  we have

$$\begin{aligned} R &= \sum w_j r_j ; RA = \frac{\sum w_j r_j / (1+r_j)}{\sum w_j / (1+r_j)} ; H' = \frac{1}{\sum w_j / r_j} ; \\ a) R - RA &= \sum w_j r_j - \frac{\sum w_j r_j / (1+r_j)}{\sum w_j / (1+r_j)} = \left[ \frac{\text{set}}{s_j = 1+r_j} \right] = \\ &= \sum w_j (s_j - 1) - \frac{\sum w_j (s_j - 1) / s_j}{\sum w_j / s_j} = \sum w_j s_j - 1 - \frac{1 - \sum w_j / s_j}{\sum w_j / s_j} = \\ &= \frac{\sum w_j s_j \sum w_j / s_j - 1}{\sum w_j s_j} \geq \left[ \text{According to Cauchy-} \right. \\ &\quad \left. \text{Schwarz inequality} \right] \geq \\ &\geq \frac{(\sum \sqrt{w_j s_j} \sqrt{w_j / s_j})^2 - 1}{\sum w_j s_j} = 0 \quad \ast \quad R \geq RA . \end{aligned} \quad (34)$$

Equality occurs when all  $\frac{\sqrt{w_j s_j}}{\sqrt{w_j / s_j}} = s_j = 1 + r_j$

are equal  $\ast$  all  $r_j$  are equal.

$$\begin{aligned}
b) \quad \frac{1}{H'(\vec{w}, \vec{r})} &= [\vec{t} = \{1/r_j\}] = R(\vec{w}, \vec{t}) \geq \\
[\text{according to a)}] &\geq RA(\vec{w}, \vec{t}) = \frac{1}{RA(\vec{w}, \vec{r})} \\
&\rightarrow H'(\vec{w}, \vec{r}) \leq RA(\vec{w}, \vec{r})
\end{aligned} \tag{35}$$

The whole theorem 9.1.5 is thus proved.

Proof of 9.2.1

$$\begin{aligned}
R &= R(r_1, \dots, r_N) = \sum w_j r_j, \\
G &= G(r_1, \dots, r_N) = \prod r_j^{w_j}, \\
RA &= RA(r_1, \dots, r_N) = \frac{1}{\sum w_j / (1+r_j)} - 1,
\end{aligned} \tag{36}$$

$$RH = RH(r_1, \dots, r_N) = \sqrt{\frac{\sum w_j r_j}{\sum w_j / r_j}};$$

We expand these functions by a Taylor approximation around  $\vec{r}_0 = \vec{1} = (1, \dots, 1)$ . In general we have the following second-order approximation, where

$$\vec{1} + \vec{\varepsilon} = (1+\varepsilon_1, \dots, 1+\varepsilon_N);$$

$$f(\vec{r}) \approx f(\vec{1}) + \sum f'_j(\vec{1}) \varepsilon_j + \frac{1}{2} \sum \sum f''_{ij}(\vec{1}) \varepsilon_i \varepsilon_j \tag{37}$$

By differentiating the respective functions we get:

$$R(\vec{1}) = G(\vec{1}) = RA(\vec{1}) = RH(\vec{1}) = 1;$$

$$R'_k(\vec{1}) = G'_k(\vec{1}) = RA'_k(\vec{1}) = RH'_k(\vec{1}) = w_k;$$

$$R''_{kk}(\vec{1}) = 0; G''_{kk}(\vec{1}) = RA''_{kk}(\vec{1}) = RH''_{kk}(\vec{1}) = -w_k(1-w_k);$$

For  $k \neq l$  we get:

$$R''_{kl}(\vec{1}) = 0; G''_{kl}(\vec{1}) = RA''_{kl}(\vec{1}) = RH''_{kl}(\vec{1}) = w_k w_l;$$

After some algebra the proof is completed.

Proof of 9.2.2

The proof is completely analogous to that of 9.2.1. We have:

$$H' = H'(r_1, \dots, r_N) = \frac{1}{\sum w_j / r_j};$$

$$H'(\vec{1}) = 1; (H')'_k(\vec{1}) = w_k; (H')''_{kk}(\vec{1}) = -2w_k(1-w_k); \quad (38)$$

$$\text{For } k \neq 1 \text{ } (H')''_{kl} = 2w_k w_l .$$

After some algebra the proof is completed.

Proof of 9.2.3

We use the same symbols as in 9.2.1 above. In general we have the following third-order approximation:

$$f(\vec{r}) \approx f(\vec{1}) + \sum_j f'_j(\vec{1}) \varepsilon_j + \frac{1}{2} \sum_i \sum_j f''_{ij}(\vec{1}) \varepsilon_i \varepsilon_j +$$

$$+ \frac{1}{6} \sum_h \sum_i \sum_j f'''_{hij}(\vec{1}) \varepsilon_h \varepsilon_i \varepsilon_j . \quad (39)$$

According to 9.2.1 RA and G have the same second-order approximations. By differentiating once more, we get:

$$G'''_{kkk}(\vec{1}) = w_k(1-w_k)(2-w_k); G'''_{kk1}(\vec{1}) = -w_k w_1(1-w_k) \text{ for } k \neq 1;$$

$$G'''_{klm}(\vec{1}) = w_k w_l w_m \text{ for } k \neq 1, l \neq m \text{ and } k \neq m.$$

$$RA'''_{kkk}(\vec{1}) = 1.5w_k(1-w_k)^2; RA'''_{kk1}(\vec{1}) = -w_k w_1(1.5w_k - 1) \text{ for } k \neq 1;$$

$$RA'''_{klm}(\vec{1}) = 1.5w_k w_l w_m \text{ for } k \neq 1, l \neq m \text{ and } k \neq m. \quad (40)$$

After some algebra the proof is completed.

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